MA 241-050 Test 2 Version 1 No Work = No Credit

1. (10 points) Integrate $\int \sin^4 x \cos^3 x \, dx$

2. (12 points) Evaluate $\int_1^{e^{\frac{\pi}{4}}} \frac{sec^4(\ln(x))}{x} \, dx$ $\left(\text{The upperbound is e raised to } \frac{\pi}{4}\right)$

3. (14 points) Use trigonometric substitution to integrate $\int \frac{2x+3}{\sqrt{9-4x^2}} \, dx$

Hint: For integrals with $\sqrt{a^2-u^2}$, use $u = a\sin\theta$

4. (10 points) Use the tables below to integrate $\int 3e^{2x} \tan^{-1}(e^x) dx$

T1: $\int \tan^{-1}u \, du = u\tan^{-1}u - \frac{1}{2}\ln(1+u^2) + C$

T2: $\int u\tan^{-1}u \, du = \frac{u^2+1}{2}\tan^{-1}u - \frac{u}{2} + C$

T3: $\int u^n \tan^{-1}u \, du = \frac{1}{n+1}\left[u^{n+1}\tan^{-1}u - \int \frac{u^{n+1}}{1+u^2} du\right] + C$

5. (12 points) Find $\int \frac{7x+12}{x^2(x+4)} dx$

6. (22 points) Determine if the following integrals are convergent or divergent. Evaluate the integral if it is convergent.

a) $\int_0^\infty e^{-2x}\cos(3x)dx$ Hint: $\int e^{ax}\cos(bx)dx = \frac{e^{ax}(a\cos(bx)+b\sin(bx))}{a^2+b^2} + C$

b) If it is possible, find the area bounded by $y = \frac{1}{\sqrt[3]{x}}$, the x-axis, the y-axis, and the line $x = 4$

7. (20 points)

a) Approximate $\int_1^9 \ln(1+x) \, dx$ using Simpson's Rule with $n = 4$

b) Find the error estimate for this approximation

Hint: $|E_s| \le \frac{K(b-a)^5}{180n^4}$ where $|f^{(4)}(x)| \le K$ for $a \le x \le b$

C2 T2 Solutions

1. (10 points)

$$\int \sin^4 x \cos^3 x \, dx = \int \sin^4 x \cos^2 x \cos x \, dx$$

$$= \int \sin^4 x (1-\sin^2 x) \cos x \, dx \qquad u = \sin x$$
$$du = \cos x \, dx$$

$$= \int u^4(1-u^2)\,du = \int u^4 - u^6 \, du$$

$$= \frac{1}{5}u^5 - \frac{1}{7}u^7 + C = \boxed{\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + C}$$

2. (12 points) $\displaystyle\int_1^{e^{\pi/4}} \frac{\sec^4(\ln x)}{x}\,dx \qquad u = \ln x$

$$du = \frac{1}{x}\,dx$$

$$\int_0^{\pi/4} \sec^4 u \, du = \int_0^{\pi/4} \sec^2 u \sec^2 u \, du$$

$$= \int_0^{\pi/4} (1+\tan^2 u)\sec^2 u \, du$$

$w = \tan u \quad dw = \sec^2 u \, du$

$$= \int_0^1 (1+w^2)\,dw = w + \frac{1}{3}w^3 \Big|_0^1$$

$$= 1 + \frac{1}{3} = \boxed{\frac{4}{3}}$$

3. (14 points)

$$\int \frac{2x+3}{\sqrt{9-\underbrace{4x^2}_{(2x)^2}}}\,dx$$

$u = (2x)$

$du = 2\,dx$

$\frac{1}{2}\,du = dx$

$$= \frac{1}{2}\int \frac{u+3}{\sqrt{9-u^2}}\,du$$

$u = 3\sin\theta$

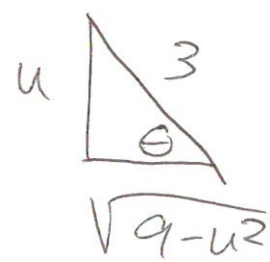
$du = 3\cos\theta\,d\theta$

$$= \frac{1}{2}\int \frac{3\sin\theta + 3}{\cancel{\sqrt{9-9\sin^2\theta}}}\,\cancel{3\cos\theta}\,d\theta$$

$$= \frac{1}{2}\int 3\sin\theta + 3\,d\theta$$

$$= \frac{1}{2}\left[-3\cos\theta + 3\theta\right] + C$$

$\frac{u}{3} = \sin\theta$



$$= \frac{1}{2}\left[-3\,\frac{\sqrt{9-u^2}}{3} + 3\sin^{-1}\left(\frac{u}{3}\right)\right] + C$$

$$= \boxed{\frac{1}{2}\left[-\cancel{3}\,\frac{\sqrt{9-4x^2}}{\cancel{3}} + 3\sin^{-1}\left(\frac{2x}{3}\right)\right] + C}$$

4. (10 points) $u = e^x \quad du = e^x\,dx$

$$\int 3u \tan^{-1} u \, du$$

$$= 3\left(\frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2}\right) + C$$

$$= \boxed{3\left(\frac{e^{2x}+1}{2} \tan^{-1} e^x - \frac{e^x}{2}\right) + C}$$

5. (12 points) $\displaystyle \int \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+4} \, dx$

$$Ax(x+4) + B(x+4) + Cx^2 = 7x + 12$$

$$Ax^2 + 4Ax + Bx + 4B + Cx^2 = 7x + 12$$

$A + C = 0$

$4A + B = 7$

$4B = 12 \quad B = 3 \qquad A = 1 \quad C = -1$

$$= \int \frac{1}{x} + \frac{3}{x^2} - \frac{1}{x+4} \, dx$$

$$= \boxed{\ln|x| - \frac{3}{x} - \ln|x+4| + C}$$

6. (22 pts)

a) $\int_0^\infty e^{-2x}\cos 3x\,dx = \lim_{t\to\infty}\int_0^t e^{-2x}\cos 3x\,dx$

$$= \lim_{t\to\infty} \frac{e^{-2x}(-2\cos 3x + 3\sin 3x)}{2^2+3^2}\Bigg|_0^t$$

$$= \lim_{t\to\infty} \frac{e^{-2t}(-2\cos 3t + 3\sin 3t)}{13} - \left(\frac{-2}{13}\right)$$

$$= 0 + \frac{2}{13} = \boxed{\frac{2}{13}}$$ converges

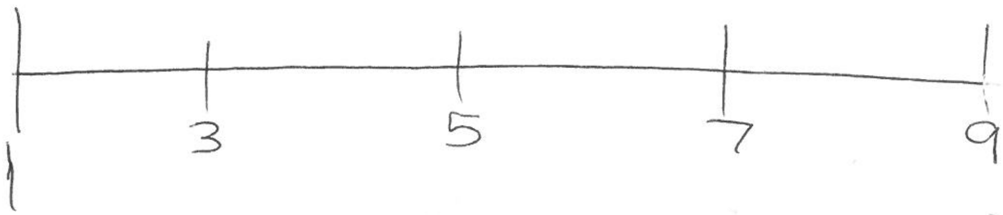
b) $\int_0^8 \frac{1}{\sqrt[3]{x}}\,dx = \lim_{t\to 0^+}\int_t^8 x^{-1/3}\,dx$

$$= \lim_{t\to 0^+} \frac{3}{2}x^{2/3}\Big|_t^8 = \lim_{t\to 0^+} \frac{3}{2}\cdot 8^{2/3} - \frac{3}{2}t^{2/3}$$

$$= \frac{3}{2}\cdot 4 = \boxed{6}$$ converges

7. (20 pts)

a) $\Delta x = \frac{b-a}{n} = \frac{9-1}{4} = 2$



$$\frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)\right]$$

$$= \boxed{\frac{2}{3}\left[\ln 2 + 4\ln(4) + 2\ln(6) + 4\ln 8 + \ln 10\right]}$$

b) $f = \ln(1+x)$

$f' = \frac{1}{1+x} = (1+x)^{-1}$

$f'' = -(1+x)^{-2}$

$f''' = 2(1+x)^{-3}$

$f^4 = -6(1+x)^{-4} = \frac{-6}{(1+x)^4}$

$$|E_S| \le \frac{\frac{6}{2^4}(9-1)^5}{180 \cdot 4^4}$$