MA 242 Test 3 Version 1

1. (18 points) Find the average value of $f(x,y) = x$ over the region D where D is bounded by the circle $x^2 + y^2 = 9$, the lines $y = x$, $y = -x$, where $x \geq 0$

2. (12 points) Set up the iterated integral to find the mass of the triangular lamina with vertices (1,2), (1,4), and (5,2) with density $\sigma(x,y) = xy$. **Do not evaluate!**

3. (18 points) Use spherical coordinates to find the mass of the solid E bounded by the hemisphere $z = \sqrt{16 - x^2 - y^2}$ and the xy - plane with density $\sigma(x,y,z) = z\sqrt{x^2 + y^2 + z^2}$

4. (20 points) Switch the order of integration

a) $\int_0^1 \int_{4x}^4 f(x,y)\,dy\,dx$

b) $\int_0^3 \int_{y^2}^9 f(x,y)\,dx\,dy$

5. (32 points) E is the solid bounded by the plane $z = 50$ and the paraboloid $z = 2x^2 + 2y^2$

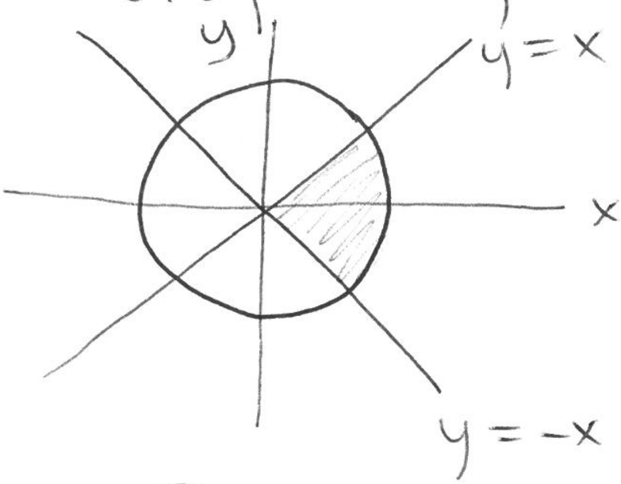
a) Sketch the graph of E

b) Set up an iterated triple integral to find the volume of E as an x - simple region. **Do not evaluate!**

c) Find the volume of E using cylindrical coordinates

C3 T3 V1 Solutions

1. (18 points)



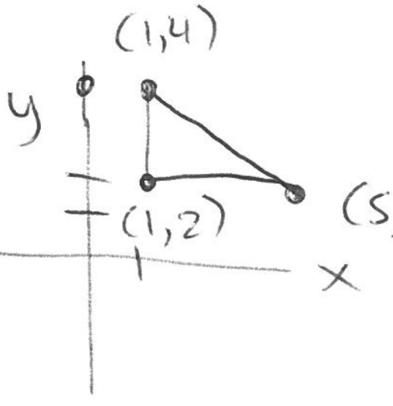
$$\frac{\iint_D x\,dA}{\iint_D 1\,dA} = \frac{\int_{-\pi/4}^{\pi/4}\int_0^3 r\cos\theta \, r\,dr\,d\theta}{\frac{1}{4}\pi(3)^2}$$

$$= \frac{\int_{-\pi/4}^{\pi/4}\cos\theta\,d\theta \int_0^3 r^2\,dr}{\frac{9\pi}{4}} = \frac{2\sin\theta\Big|_0^{\pi/4}\ \frac{1}{3}r^3\Big|_0^3}{\frac{9\pi}{4}}$$

$$= \frac{\cancel{2}(\sqrt{2}/\cancel{2})\ \cancel{9}}{\frac{\cancel{9}\pi}{4}} = \boxed{\frac{4\sqrt{2}}{\pi}}$$

2. (12 points)

$$\boxed{\int_1^5 \int_2^{4.5-\frac{x}{2}} xy\,dy\,dx}$$

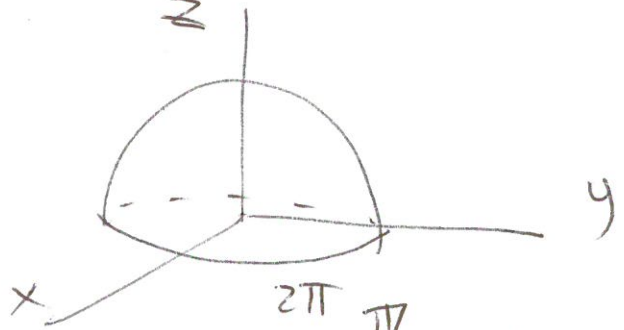


$m = \frac{\Delta y}{\Delta x} = \frac{2-4}{5-1} = \frac{-2}{4}$

$y = -\frac{1}{2}x + b$

$4 = -\frac{1}{2} + b \quad b = 4.5$

3. (18 points)



$$m = \int_0^{2\pi}\int_0^{\pi/2}\int_0^4 \rho\cos\phi \sqrt{\rho^2}\, \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$2\pi \int_0^{\pi/2} \cos\phi \sin\phi \int_0^4 \rho^4 \, d\rho$$

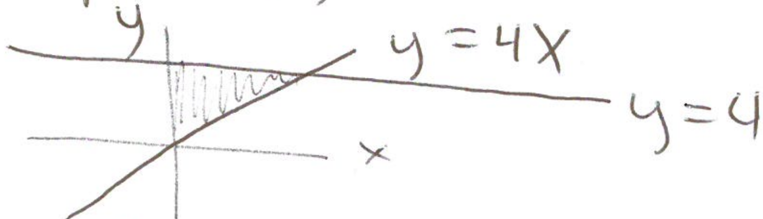
$u = \sin\phi$
$du = \cos\phi \, d\phi$

$$2\pi \int_0^1 u \, du \ \tfrac{1}{5}\rho^5 \Big|_0^4$$

$$2\pi \ \tfrac{1}{2}u^2\Big|_0^1 \ \tfrac{1}{5}\cdot 4^5 = \boxed{\tfrac{\pi}{5}\cdot 4^5}$$

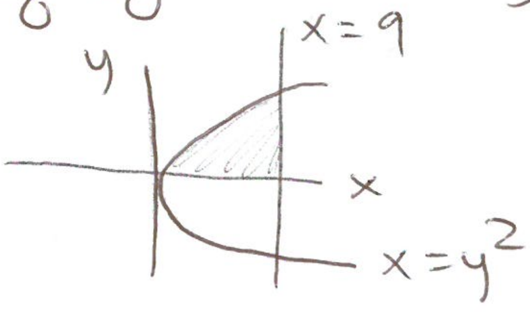
4. (20 points)

a)



$$\int_0^4 \int_0^{\frac{y}{4}} f(x,y)\, dx\, dy$$

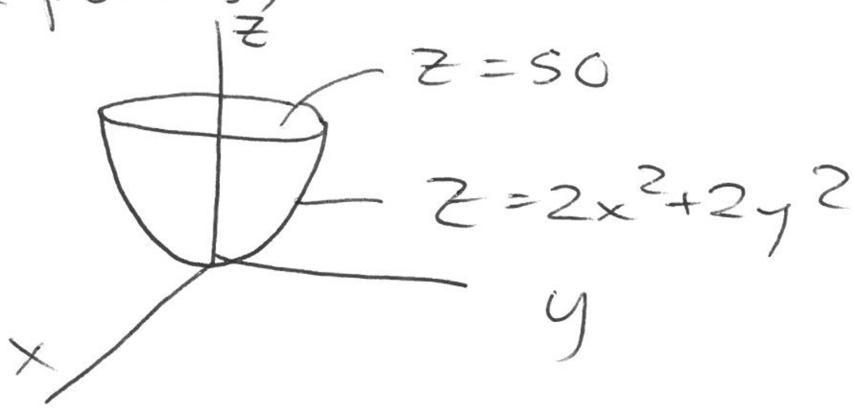
b)



$$\int_0^9 \int_0^{\sqrt{x}} f(x,y)\, dy\, dx$$

5. (32 points)

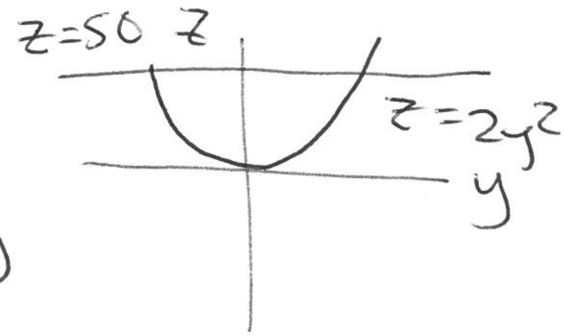
a)



b)

$$\frac{z-2y^2}{2} = x^2$$

$$\int_{-5}^{5}\int_{2y^2}^{50}\int_{-\sqrt{\frac{z}{2}-y^2}}^{\sqrt{\frac{z}{2}-y^2}} 1 \, dx\, dz\, dy$$



c) $$\int_0^{2\pi}\int_0^{5}\int_{2r^2}^{50} 1\, dz\, r\, dr\, d\theta$$

$$50 = 2r^2$$

$$25 = r^2$$

$$= 2\pi \int_0^5 z \Big|_{2r^2}^{50} r\, dr$$

$$= 2\pi \int_0^5 (50-2r^2)\, r\, dr$$

$$= 2\pi \int_0^5 50r - 2r^3\, dr$$

$$2\pi\left[25r^2 - \frac{2}{4}r^4\right]_0^5 = 2\pi\left[25^2 - \frac{1}{2}25^2\right] = \boxed{\pi \cdot 25^2}$$