MA 242 Test 4 Version 1

1. (15 points) Find the line integral with respect to arc length of $f(x,y,z) = zx + y$ along the line segment from (1,3,2) to (6,-9,2).

2. (21 points) Use the vector field $\vec{\mathbf{F}}(x,y,z) = \langle zx^4, \ln(x)y, x+2 \rangle$ to answer the following

a) Show that $\vec{\mathbf{F}}(x,y,z)$ is **not** path independent

b) Find the work done by the moving a particle along the curve $x = e^y$ in the plane $z = 10$ from $(1,0,10)$ to $(e^3,3,10)$

3. (16 points) Find the surface area of the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the plane $z = 3$

4. (28 points) Use $\vec{\mathbf{F}}(x,y,z) = \left(\frac{1}{\sqrt{x}} + \sin(\pi y)\right)\mathbf{i} + \left(\pi x\cos(\pi y) + 3y^2\right)\mathbf{j} + (4)\mathbf{k}$ to answer the following.

a) Assume $\vec{\mathbf{F}}(x,y,z)$ is conservative, find its most general potential function f

b) Find the work done by $\vec{\mathbf{F}}(x,y,z)$ on a particle that moves along

$\vec{r}(t) = \langle 4t-3, (t-1)(t-3), t-1 \rangle \ 1 \le t \le 3$

5. (10 points) Find a parametric representation for the half circle $x^2 + y^2 = 4$, $x \ge 0$ from (0,-2) to (0,2)

6. (10 points) a) Find the gradient vector field of $f(x,y) = y^2 - x^2$

b) Describe the direction of vectors in the first quadrant

C3T4V1 Solutions

1. (15 points) $\int_C f\,ds$

$\vec{r}(t) = \vec{r}_0(1-t) + \vec{r}_1 t = \langle 1,3,2\rangle(1-t) + \langle 6,-9,2\rangle t$

$= \langle 1-t, 3-3t, 2-2t\rangle + \langle 6t, -9t, 2t\rangle = \langle 1+5t, 3-12t, 2\rangle$

$\vec{r}\,'(t) = \langle 5, -12, 0\rangle$

$\int_0^1 \left(2(1+5t) + (3-12t)\right)\sqrt{5^2+144}\,dt$

$= \int_0^1 (2 + 10t + 3 - 12t)\sqrt{169}\,dt$

$= \int_0^1 (5-2t)\,13\,dt = (5t - t^2)\Big|_0^1 13 = 4(13)$

$= \boxed{52}$

2. (21 points)

a) $\text{curl}\,\vec{F} = \nabla\times\vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ zx^4 & \ln(x)y & x+z \end{vmatrix} = \langle 0-0, -(1-x^4), \frac{y}{x}-0\rangle$

$\neq \vec{0}$ so not conservative & not path indep.

b) $x = e^t$, $y = t$, $z = 10$

$\vec{r} = \langle e^t, t, 10\rangle$

$\vec{r}\,' = \langle e^t, 1, 0\rangle$

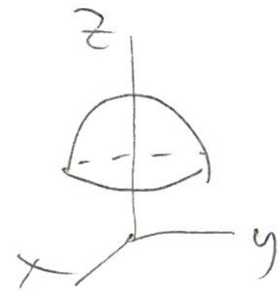
$\int_C \vec{F}\cdot d\vec{r} = \int_0^3 \langle 10e^{4t}, t^2, e^t+2\rangle\cdot\langle e^t, 1, 0\rangle\,dt$

$= \int_0^3 10e^{5t} + t^2\,dt = 2e^{5t} + \frac{1}{3}t^3\Big|_0^3 = 2e^{15} + 9 - 2e^0$

$\boxed{2e^{15}+7}$

3. (16 points)

$$SA = \iint_D \sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}\ dA$$



$$= \iint_D \sqrt{(-2x)^2 + (-2y)^2 + 1}\ dA$$

$$= \iint_D \sqrt{4x^2 + 4y^2 + 1}\ dA$$

$4 - r^2 = 3$

$1 = r^2$

$$= \int_0^{2\pi}\int_0^1 \sqrt{4r^2+1}\ r\,dr\,d\theta$$

$u = 4r^2 + 1$

$du = 8r\,dr$

$\frac{1}{8} du = r\,dr$

$$= 2\pi \frac{1}{8} \int_1^5 \sqrt{u}\ du$$

$$= \frac{\pi}{4} \cdot \frac{2}{3} u^{3/2} \Big|_1^5$$

$$= \boxed{\frac{\pi}{6}\left[5^{3/2} - 1\right]}$$

4. (28 points)

a) $f_x = x^{-1/2} + \sin \pi y \quad f_y = \pi x \cos \pi y + 3y^2 \quad f_z = 4$

$f = 2x^{1/2} + x \sin \pi y + g(y,z)$

$f_y = 0 + \pi x \cos \pi y + g_y(y,z) =$

$g_y = 3y^2$

$g = y^3 + h(z)$

$f = 2x^{1/2} + x\sin \pi y + y^3 + h(z)$

$f_z = 0 + h'(z) = 4 \qquad h(z) = 4z + K$

$$\boxed{f = 2\sqrt{x} + x \sin \pi y + y^3 + 4z + K}$$

4b) $f = 2\sqrt{x} + x\sin\pi y + y^3 + 4z + k$

$\vec{r}(1) = \langle 1, 0, 0 \rangle$

$\vec{r}(3) = \langle 9, 0, 2 \rangle$

$$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$$

$$= f(9,0,2) - f(1,0,0)$$

$$= 2\sqrt{9} + \cancel{9\sin 0} + 0^3 + 4(2) - [2]$$

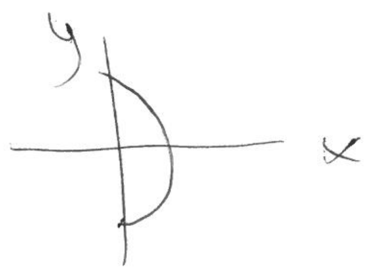
$$= 6 + 8 - 2 = \boxed{12}$$

5. (10 points)

$x = 2\cos t$

$y = 2\sin t$

$-\frac{\pi}{2} \le t \le \frac{\pi}{2}$



6. (10 points)

a) $\nabla f = \langle -2x, 2y \rangle$

b) they point up & to the left