

MA 341 601 – Fall 2025 – KURTZ Exam #1

Student's Name: _____ **(print neatly)**

Date of Exam: Thursday Sept 11, 2025 - Monday, Sept 15, 2025 **Time Limit:** 60 minutes

Allowed Materials: None (closed book & closed notes)

Student – NC State University Pack Pledge:

I, _____ (student's printed name) have neither given nor received unauthorized aid on this exam or assignment. I have read the instructions and acknowledge that this is the correct exam.

1. (26 points) Use the Initial Value Problem (IVP): $\frac{dy}{dx} = \frac{\sin x}{2y \cos^2 x}$, $y(0) = -3$ to answer the following:

a) Solve the IVP. Write your answer with y as an explicit function of x if possible.

b) Does the Existence and Uniqueness Theorem guarantee that this IVP has a unique solution? Justify your answer.

2. (20 points) A large tank initially contains 30 liters of pure water. Brine solution flows into the tank at a rate of 6 L/min. The well-mixed solution leaves the tank at a rate of 4 L/min. If the concentration of salt in the brine entering the tank is 0.5 kg/L and $x(t)$ is the amount of salt in the tank at time t , find $x(t)$.

3. (14 points) Use $(x^2 + bxy)dx + (3x^2 + \sec^2 y) dy = 0$ to answer the following:
- Find the value of b that makes this an exact differential equation
 - Solve the differential equation using that value of b .

4. (12 points) Solve the Boundary Value Problem (BVP):
 $y'' - 8y' + 16y = 0$, $y(0) = 2$, $y(1) = 0$

5. (15 points) Use the differential equation $\frac{dy}{dt} = (9 - y)(y + 2)$ to answer the following:
- a) Sketch its phase line and classify its equilibria as we have done in class
 - b) If $y(0)=9$, without solving the differential equation, find $y(50)$.
 - c) Use the phase line to determine the asymptotic behavior as $t \rightarrow \infty$ of the solution through $y(0)= 1$
 - d) Is this differential equation separable, linear, both, or neither? No explanation is needed.

6. (13 points) Determine if $y + e^{xy} = x$ is an implicit solution to $\frac{dy}{dx} = \frac{e^{-xy} - y}{e^{-xy} + x}$