MA 341 T1 Solutions

1. (26 points)

a) $\int 2y\,dy = \int \frac{\sin x}{\cos^2 x}\,dx \qquad u = \cos x \quad du = -\sin x\,dx$

$y^2 = \int -\frac{1}{u^2}\,du$

$= \frac{1}{u} + C$

$y = \pm\sqrt{\frac{1}{u} + C}$

$y = \pm\sqrt{\frac{1}{\cos x} + C}$

$y(0) = -3 = -\sqrt{\frac{1}{\cos 0} + C} \qquad C = 8$

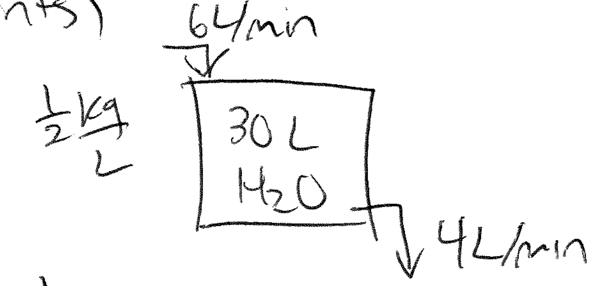
$\boxed{y = -\sqrt{\frac{1}{\cos x} + 8}}$ or $y = -\sqrt{\sec x + 8}$

b) $f = \frac{\sin x}{2y\cos^2 x}$ f is cont at & around $(0,-3)$

$\frac{\partial f}{\partial y} = \frac{\sin x}{2\cos^2 x}\left(\frac{-1}{y^2}\right) \qquad \frac{\partial f}{\partial y}$ is cont at & around $(0,-3)$

Yes

2. (20 points)



$$\frac{dx}{dt} = F_i C_i - F_o C_o$$
$$= 6\left(\tfrac{1}{2}\right) - 4\left(\frac{x}{30+2t}\right)$$

$$x(0) = 0$$

$$\frac{dx}{dt} = 3 - \frac{4x}{2(15+t)} = 3 - \frac{2x}{15+t}$$

$$\frac{dx}{dt} + \frac{2x}{15+t} = 3$$

$$\mu = e^{\int \frac{2}{15+t}\,dt} = e^{2\ln(15+t)} = e^{\ln(15+t)^2} = (15+t)^2$$

$$\int \frac{d}{dt}\left[(15+t)^2 x\right] = \int 3(15+t)^2$$

$$(15+t)^2 x = (15+t)^3 + C$$

$$x = \frac{(15+t)^3 + C}{(15+t)^2}$$

$$x(0) = 0 \rightarrow C = -15^3$$

$$\boxed{x = \frac{(15+t)^3 - 15^3}{(15+t)^2}}$$

3. (14 points)

a) $M_y = 6x$ $\quad N_x = 6x$

$\boxed{b=6}$

b) $F_x = x^2 + 6xy$ $\quad F_y = 3x^2 + \sec^2 y$

$F = \frac{1}{3}x^3 + 3x^2y + g(y)$

$F_y = 0 + 3x^2 + g'(y) =$

$g(y) = \tan y$

$$\boxed{\frac{1}{3}x^3 + 3x^2y + \tan y = C}$$

4. (12 points)

$r^2 - 8r + 16 = 0$

$(r-4)^2 = 0$

$y = c_1e^{4t} + c_2te^{4t}$

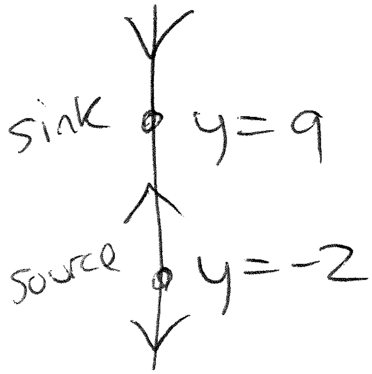
$y(0) = 2 = c_1e^0 + c_2 \cdot 0e^0$ $\quad c_1 = 2$

$y(1) = 0 = 2e^4 + c_2 \cdot 1e^4$ $\quad c_2 = -2$

$$\boxed{y = 2e^{4t} - 2te^{4t}}$$

5. (15 points)

a) $y > 9$



b) $y(50) = 9$

c) $y \to 9$

d) separable

6. (13 points)

$$\frac{dy}{dx} + e^{xy}\left(y + x\frac{dy}{dx}\right) = 1$$

$$\frac{dy}{dx} + e^{xy}y + xe^{xy}\frac{dy}{dx} = 1$$

$$\frac{dy}{dx}\left(1 + xe^{xy}\right) = 1 - e^{xy}y$$

$$\frac{dy}{dx} = \frac{1 - e^{xy}y}{1 + xe^{xy}} \cdot \frac{e^{-xy}}{e^{-xy}} = \frac{e^{-xy} - y}{e^{-xy} + x} \checkmark$$

Yes