1. (25 points) Find a particular solution to $\vec{\boldsymbol{x}}' = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix} \vec{\boldsymbol{x}} + \begin{bmatrix} 3e^{2t} \\ -6e^{2t} \end{bmatrix}$ using the **method of variation of parameters** if $\vec{\boldsymbol{x}}_c = c_1 e^{2t} \begin{bmatrix} -2 \\ 1 \end{bmatrix} + c_2 e^{5t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

Hint: $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad-bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

2. (16 points) Find the general solution $\vec{\boldsymbol{x}}\,' = \begin{bmatrix} 4 & -3 & -2 \\ 0 & 7 & 2 \\ 0 & 0 & 4 \end{bmatrix} \vec{\boldsymbol{x}}$

3. (15 points) Tank A initially holds 100 L of pure water; tank B initially holds 50 L of a brine solution containing 2 kg of dissolved salt. The tanks are connected by pipes. The liquid in tank A flows into tank B at a rate of 10 L/min and from tank B into tank A at a rate of 6 L/min. A solution containing 0.1 kg/L of salt is poured into tank A at a rate of 5 L/min and pure water enters tank B at a rate of 2 L/min. Both tanks are well -mixed. The contents of tank A flow out of a drain at the bottom of tank A at a rate of 1 L/min and the contents of tank B flow out of a drain at the bottom of tank B at a rate of 6 L/min.

If $x_1(t)$ is the amount of salt in tank A and $x_2(t)$ is the amount of salt in tank B, find the coefficient matrix **A**, $\vec{\mathbf{f}}$, and $\vec{\boldsymbol{x}}$ (0) so that $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \mathbf{A} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \vec{\mathbf{f}}$

Do not solve this system!

4. (15 points) Use $\vec{\boldsymbol{x}}' = \begin{bmatrix} 4 & 1 \\ -5 & 0 \end{bmatrix} \vec{\boldsymbol{x}} + \begin{bmatrix} -4t + 3 \\ 5t \end{bmatrix}$ to find its complementary solution $\vec{\boldsymbol{x}}_c$

Hint: If r=$\alpha + \beta i$ with **u**=**a** + **b**i then two linearly independent solutions are $e^{\alpha t}(cos(\beta t)\mathbf{a} - \sin(\beta t)\mathbf{b})$ and $e^{\alpha t}(cos(\beta t)\mathbf{b} + \sin(\beta t)\mathbf{a})$

5. (17 points) Use $\vec{\boldsymbol{x}}' = \begin{bmatrix} 4 & 1 \\ -5 & 0 \end{bmatrix} \vec{\boldsymbol{x}} + \begin{bmatrix} -4t+3 \\ 5t \end{bmatrix}$ to find its particular solution $\vec{\boldsymbol{x}}_p$ using the **method of undetermined coefficients**. You don't need to solve problem 4 to do problem 5.

6. (12 points) Find the inverse of $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 2 \\ 0 & 3 & 1 \end{bmatrix}$ using row operations