MA 341 T2 Solutions

1. (15 points)

$r^2 + 9 = 0$

$y_c = C_1 \cos 3t + C_2 \sin 3t$

$y_p = Ae^{2t} \quad y_p' = 2Ae^{2t} \quad y_p'' = 4Ae^{2t}$

$4Ae^{2t} + 9Ae^{2t} = 13e^{2t}$

$A = 1$

$y = C_1 \cos 3t + C_2 \sin 3t + 1e^{2t}$

$y(0) = 0 = C_1 \cos 0 + \underbrace{\cancel{C_2 \sin 0}}^{0} + e^0$

$0 = C_1 + 1 \qquad C_1 = -1$

$y = -\cos 3t + C_2 \sin 3t + e^{2t}$

$y' = 3\sin 3t + 3C_2 \cos 3t + 2e^{2t}$

$y'(0) = 8 = 3C_2 + 2 \qquad C_2 = 2$

$$\boxed{y = -\cos 3t + 2\sin 3t + e^{2t}}$$

2. (14 points)

$$r^2-2r+1=0$$
$$(r-1)^2=0$$
$$y_c=c_1e^t+c_2te^t$$

$$y_p=v_1e^t+v_2te^t$$

$$-\left(v_1'e^t+v_2'te^t=0\right)$$
$$v_1'e^t+v_2'(1e^t+te^t)=\frac{e^t}{t^2+1}$$

$$v_2'e^t=\frac{e^t}{t^2+1}$$

$$v_2'=\frac{1}{t^2+1} \qquad v_2=\tan^{-1}t$$

$$v_1'=-v_2't=-\frac{t}{t^2+1}$$

$$v_1=\int\frac{-t}{t^2+1}\,dt \qquad u=t^2+1 \qquad du=2t\,dt$$
$$\tfrac{1}{2}du=t\,dt$$
$$=\int-\frac{1}{2}\frac{1}{u}\,du=\frac{-1}{2}\ln(t^2+1)$$

$$\boxed{y_p=-\frac{1}{2}\ln(t^2+1)e^t+(\tan^{-1}t)te^t}$$

3. (15 points) $my'' + by' + ky = F_{ext}$

a) $\boxed{3y'' + 3y' + 6y = 18\cos t \quad y(0)=1 \quad y'(0)=0}$

$F = ky$

$mg = ky$

$3(10) = k(5)$

$k = 6$

b) $y'' + y' + 2y = 6\cos t$

$y_p = A\cos t + B\sin t$

$y_p' = -A\sin t + B\cos t$

$y_p'' = -A\cos t - B\sin t$

$\underline{-A\cos t} - B\sin t - A\sin t + \underline{B\cos t} + \underline{2A\cos t} + \underbrace{2B\sin t}_{} = 6\cos t$

$A + B = 6$

$B - A = 0$

$A = B = 3$

$\boxed{y_p = 3\cos t + 3\sin t}$

4. (12 points) $\mathcal{L}\{2e^{4t}\} = \int_0^\infty e^{-st} 2e^{4t}\,dt$

$= \lim_{n\to\infty} \int_0^n 2e^{(4-s)t}\,dt = \lim_{n\to\infty} \frac{2}{4-s} e^{(4-s)t}\Big|_0^n$

$= \lim_{n\to\infty} \frac{2}{4-s}\left[e^{(4-s)n} - e^0\right] = \frac{2}{4-s}\left[0-1\right] = \frac{-2}{4-s} = \frac{2}{s-4}$

$\boxed{s>4}$ domain

5. (12 points)

$$f = 2t - 2t\,u(t-6) + 7u(t-8)$$

$$\mathcal{L}\{f\} = \frac{2}{s^2} - 2e^{-6s}\mathcal{L}\{t+6\} + \frac{7e^{-8s}}{s}$$

$$= \frac{2}{s^2} - 2e^{-6s}\left[\frac{1}{s^2} + \frac{6}{s}\right] + \frac{7e^{-8s}}{s}$$

6. (15 points)

$$F(s) = \frac{12+7s}{s^2(s+4)} = \frac{A}{s} + \frac{B}{s^2} + \frac{C}{s+4}$$

$$As(s+4) + B(s+4) + Cs^2 = 12 + 7s$$

$$\underbrace{As^2} + \underline{4As} + \underline{Bs} + 4B + \underbrace{Cs^2} =$$

$A + C = 0$

$4A + B = 7$

$4B = 12 \quad B = 3 \quad A = 1 \quad C = -1$

$$F(s) = \frac{1}{s} + \frac{3}{s^2} - \frac{1}{s+4}$$

$$f(t) = 1 + 3t - e^{-4t}$$

$$\mathcal{L}^{-1}\{F(s)e^{-2s}\} = f(t-2)u(t-2)$$

$$\boxed{= \left[1 + 3(t-2) - e^{-4(t-2)}\right]u(t-2)}$$

7. (17 points)

$$s^2L - s\cancel{y(0)}^{0} - \cancel{y'(0)}^{0} - 4[sL - \cancel{y(0)}^{0}] + 13L = \frac{39}{s}$$

$$(s^2 - 4s + 13)L = \frac{39}{s}$$

$$L = \frac{39}{s(s^2-4s+13)} = \frac{A}{s} + \frac{B(s-2)+3C}{(s-2)^2+3^2}$$

$$(s-\alpha)^2 + \beta^2$$

$$s^2 - 2\alpha s + \alpha^2 + \beta^2$$

$$\alpha = 2$$

$$4 + \beta^2 = 13$$

$$A(s^2-4s+13) + (B(s-2)+3C)s = 39 \quad \beta = 3$$

$$As^2 - 4As + 13A + Bs^2 - 2Bs + 3Cs = 39$$

$$A + B = 0$$

$$-4A - 2B + 3C = 0$$

$$13A = 39 \quad A = 3 \quad B = -3$$

$$-12 + 6 + 3C = 0 \qquad C = 2$$

$$\boxed{y = 3 - 3e^{2t}\cos 3t + 2e^{2t}\sin 3t}$$