1. (15 points) Use **the method of undetermined coefficients** to solve the Initial Value Problem (IVP): $y'' + 9y = 13e^{2t}$; y(0)=0, $y'(0)$=8

2. (14 points) Use **the method of variation of parameters** to find a particular solution to:

$$y'' - 2y' + y = \frac{e^t}{t^2+1}$$

3. (15 points) A 3 kg mass attached to the end of a hanging spring stretches the spring 5 m upon coming to rest at equilibrium. Its damping constant is 3 Ns/m. At t=0, an external force of 18 cost N is applied to the system. Hint: If it is needed, use gravity= 10 m/s^2 to answer the following:
   a) If the mass is pulled down 1 m from its equilibrium position and released. Formulate the IVP that describes this system (**Do not solve**)

   b) Find its steady state solution

4. (12 points) Use the definition of the Laplace transform to find the Laplace transform of f(t)=$2e^{4t}$ and state its domain.

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2+b^2}$	$L\{e^{at}\} = \frac{1}{s-a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2+b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2+b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2+b^2}$
$L\{g(t)u(t-a)\} = e^{-as}L\{g(t+a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

5. (12 points) Express the given function using unit step functions and find its Laplace transform $f(t) = \begin{cases} 2t, & t < 6 \\ 0, & 6 \le t < 8 \\ 7, & 8 \le t \end{cases}$

Use the table below to answer the following:

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2 + b^2}$	$L\{e^{at}\} = \frac{1}{s - a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2 + b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s-a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s-a}{(s-a)^2 + b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s-a)^2 + b^2}$
$L\{g(t)u(t-a)\} = e^{-as}L\{g(t+a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t-a)u(t-a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t-a)\} = \frac{e^{-as}}{s}$

6. (15 points) Find the inverse Laplace of the following: $\frac{12+7s}{s^2(s+4)}e^{-2s}$

$L\{y''\} = s^2L\{y\} - sy(0) - y'(0)$	$L\{\cos(bt)\} = \frac{s}{s^2 + b^2}$	$L\{e^{at}\} = \frac{1}{s - a}$
$L\{y'\} = sL\{y\} - y(0)$	$L\{\sin(bt)\} = \frac{b}{s^2 + b^2}$	$L\{t^n\} = \frac{n!}{s^{n+1}}$
$L\{t^n e^{at}\} = \frac{n!}{(s - a)^{n+1}}$	$L\{e^{at}\cos(bt)\} = \frac{s - a}{(s - a)^2 + b^2}$	$L\{e^{at}\sin(bt)\} = \frac{b}{(s - a)^2 + b^2}$
$L\{g(t)u(t - a)\} = e^{-as}L\{g(t + a)\}$	$L^{-1}\{e^{-as}F(s)\} = f(t - a)u(t - a)$	$L\{t^n f(t)\} = (-1)^n \frac{d^n(F(s))}{ds^n}$
	$L\{1\} = \frac{1}{s}$	$L\{u(t - a)\} = \frac{e^{-as}}{s}$

7. (17 points) Use the method of Laplace transforms to solve the Initial Value Problem:
   $y'' - 4y' + 13y = 39$; $y(0)=0$, $y'(0) = 0$