

C2 T1 V1 Solutions

1. (14 points).

$$f_{ave} = \frac{1}{b-a} \int_a^b f dx$$

$$= \frac{1}{\frac{\pi}{4}-0} \int_0^{\frac{\pi}{4}} x \cos(2x) dx$$

LIATE $u = x$ $v = \frac{1}{2} \sin 2x$
 $du = dx$ $dv = \cos 2x dx$

$$= \frac{4}{\pi} \left[uv - \int v du \right]$$

$$= \frac{4}{\pi} \left[\frac{x}{2} \sin 2x \Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} \frac{1}{2} \sin 2x dx \right]$$

$$= \frac{4}{\pi} \left[\frac{\pi}{8} \sin \frac{\pi}{2} + \frac{1}{4} \cos 2x \Big|_0^{\frac{\pi}{4}} \right]$$

$$= \frac{4}{\pi} \left[\frac{\pi}{8} + \frac{1}{4} \cos \frac{\pi}{2} - \frac{1}{4} \cos 0 \right]$$

$$= \boxed{\frac{4}{\pi} \left[\frac{\pi}{8} - \frac{1}{4} \right]}$$

$$= \frac{1}{2} - \frac{1}{\pi}$$

2. (14 points)

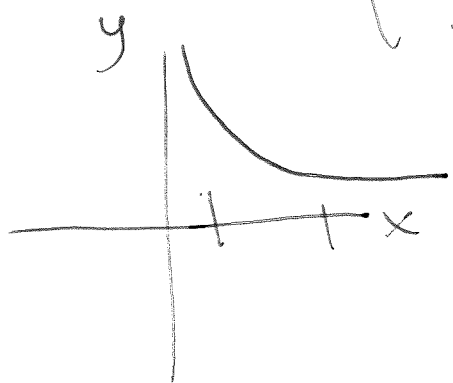
$$30 = \int_0^1 kx \, dx = \frac{k}{2} x^2 \Big|_0^1 = \frac{k}{2}$$

$$k = 60$$

$$W = \int_1^2 60x \, dx = 30x^2 \Big|_1^2 = 30[4-1] = \boxed{90 \text{ J}}$$

3. (14 points)

$$(\bar{x}, \bar{y}) = \left(\frac{\int_a^b x f(x) \, dx}{\int_a^b f(x) \, dx}, \frac{\int_a^b \frac{1}{2} [f(x)]^2 \, dx}{\int_a^b f(x) \, dx} \right)$$



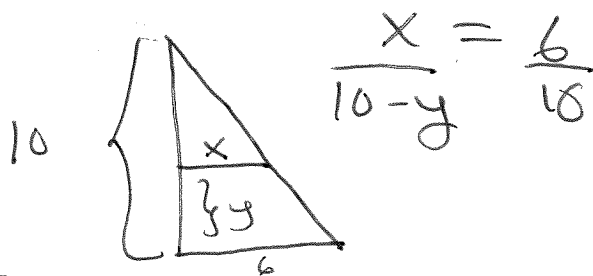
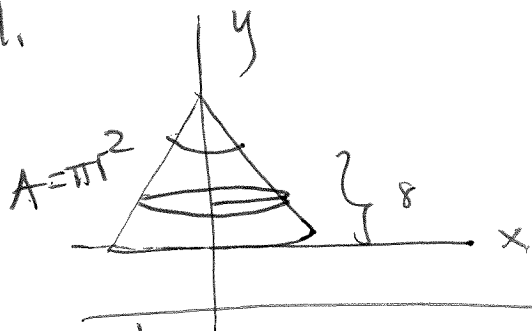
$$(\bar{x}, \bar{y}) = \left(\frac{\int_1^3 x \cdot \frac{1}{x} \, dx}{\int_1^3 \frac{1}{x} \, dx}, \frac{\int_1^3 \frac{1}{2} \cdot \frac{1}{x^2} \, dx}{\int_1^3 \frac{1}{x} \, dx} \right)$$

$$= \left(\frac{x \Big|_1^3}{\ln x \Big|_1^3}, \frac{-\frac{1}{2} \frac{1}{x} \Big|_1^3}{\ln x \Big|_1^3} \right)$$

$$= \left(\frac{3-1}{\ln 3 - \ln 1}, \frac{-\frac{1}{2} \left[\frac{1}{3} - 1 \right]}{\ln 3 - \ln 1} \right)$$

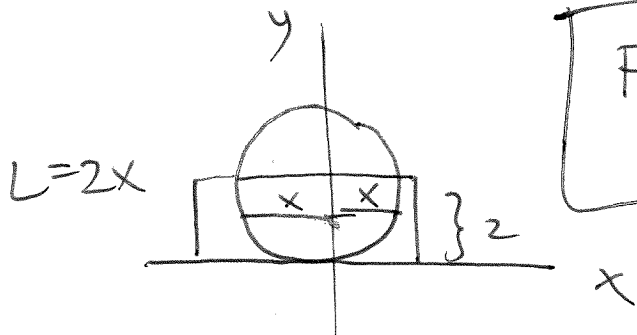
$$= \boxed{\left(\frac{2}{\ln 3}, \frac{-\frac{1}{2}(-\frac{2}{3})}{\ln 3} \right)}$$

4.



$$W = \int_0^8 62.4 \left[\pi \left(\frac{6}{10} (10-y) \right)^2 \right] (11-y) dy \text{ ft-lb}$$

5. (15 points)

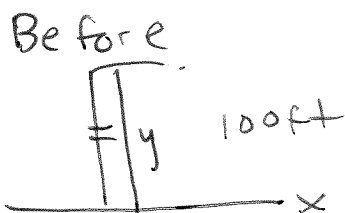


$$F = \int_0^2 1000(9.8) \left[2 \sqrt{4-(y-2)^2} \right] (2-y) dy \text{ N}$$

$$(x-0)^2 + (y-2)^2 = 2^2$$

$$x = \sqrt{4-(y-2)^2}$$

6. (14 points)



$$W_{\text{piece}} \approx F d$$

$$\frac{1}{5} \Delta y (100-y)$$

$$\frac{20 \text{ pounds}}{100 \text{ ft}} = \frac{1}{5} \text{ lb/ft}$$

$$W = \int_0^{100} \frac{1}{5} (100-y) dy = \frac{1}{5} \left(100y - \frac{1}{2} y^2 \right) \Big|_0^{100}$$

$$= \frac{1}{5} \left(\frac{100^2}{2} \right) \text{ ft-lb}$$

7. (14 points)

$$\frac{dx}{dt} = e^t - 1 \quad \frac{dy}{dt} = 2e^{t/2}$$

$$L = \int_0^2 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$$

$$= \int_0^2 \sqrt{(e^t - 1)^2 + (2e^{t/2})^2} dt$$

$$= \int_0^2 \sqrt{e^{2t} - 2e^t + 1 + 4e^t} dt$$

$$= \int_0^2 \sqrt{e^{2t} + 2e^t + 1} dt$$

$$= \int_0^2 \sqrt{(e^t + 1)^2} dt$$

$$= \int_0^2 e^t + 1 dt = e^t + t \Big|_0^2$$

$$= e^2 + 2 - e^0 = \boxed{e^2 + 1}$$