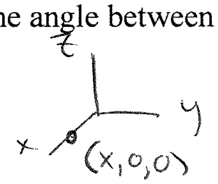
MA 242 Test 1 Version 1

Name: Solutions (print your first and last name neatly)

1. (28 points) Use points A (1,1,2) and B (3,1,3) and vector $\vec{c} = \langle 1, -2, 1 \rangle$ to answer the following:
   a) Find the distance from point A to the x-axis
   b) Find a vector in the same direction as $\vec{c}$, but with magnitude 7
   c) Find the area of the parallelogram with adjacent edges $\overrightarrow{AB}$ and $\vec{c}$
   d) Find the angle between $\overrightarrow{AB}$ and $\vec{c}$

a)



$$\sqrt{(1-1)^2 + (1-0)^2 + (2-0)^2} = \boxed{\sqrt{5}}$$

b) $$\hat{c} = \frac{\vec{c}}{\|\vec{c}\|} = \frac{\langle 1,-2,1 \rangle}{\sqrt{6}}$$

$$\boxed{7\,\frac{\langle 1,-2,1 \rangle}{\sqrt{6}}}$$

c) $$\overrightarrow{AB} = \langle 3-1,\ 1-1,\ 3-2 \rangle = \langle 2, 0, 1 \rangle$$

$$\overrightarrow{AB} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 1 \\ 1 & -2 & 1 \end{vmatrix} = \langle 0+2, -(2-1), -4 \rangle$$

$$= \langle 2, -1, -4 \rangle$$

$$\|\overrightarrow{AB} \times \vec{c}\| = \sqrt{4+1+16} = \boxed{\sqrt{21}}$$

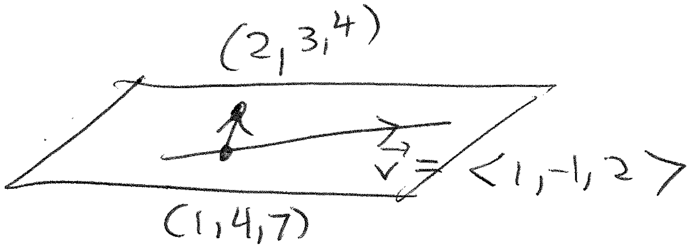
d) $$\vec{a} \cdot \vec{b} = \|\vec{a}\|\,\|\vec{b}\| \cos\theta$$

$$\langle 2,0,1 \rangle \cdot \langle 1,-2,1 \rangle = \sqrt{5}\,\sqrt{6}\,\cos\theta$$

$$\frac{3}{\sqrt{5}\sqrt{6}} = \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{3}{\sqrt{5}\sqrt{6}}\right)}$$

2. (15 points) Find an equation of the plane containing the point A(2,3,4) and the line x=1+t, y=4-t, z= 7+2t



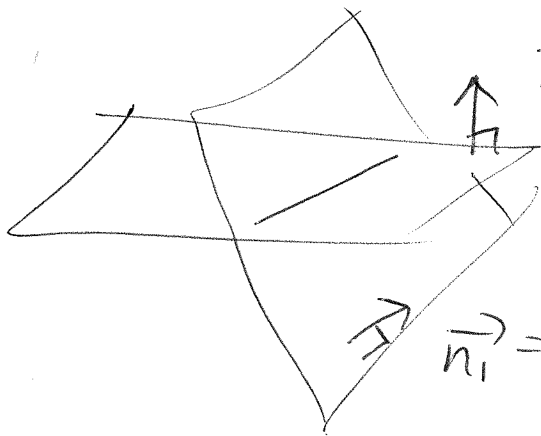
$$\langle 2-1,\ 3-4,\ 4-7 \rangle = \langle 1,-1,-3 \rangle$$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -3 \\ 1 & -1 & 2 \end{vmatrix} = \langle -2-3,\ -(2+3),\ -1+1 \rangle$$

$$= \langle -5, -5, 0 \rangle$$

$$\boxed{-5(x-2) - 5(y-3) + 0(z-4) = 0}$$

3. (15 points) Find a vector equation of the line of intersection of the planes 2x-y+3z=5 and x+y+6z=1



$\vec{n_2} = \langle 1, 1, 6 \rangle$

$\vec{n_1} = \langle 2, -1, 3 \rangle$

$$\vec{v} = \vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 3 \\ 1 & 1 & 6 \end{vmatrix}$$

$$\left.\begin{aligned} 2x - y + 3z &= 5 \\ x + y + 6z &= 1 \end{aligned}\right\} z = 0$$

$3x = 6 \quad x = 2, y = -1$

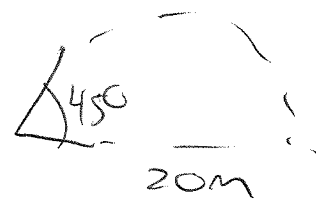
$(2, -1, 0)$

$$= \langle -6-3, -(12-3), 2+1 \rangle$$

$$= \langle -9, -9, 3 \rangle$$

$$\boxed{\vec{r} = \langle 2, -1, 0 \rangle + \langle -9, -9, 3 \rangle t}$$

4. (15 points) A ball is thrown from the ground at an angle of elevation of 45° above the horizontal with an initial speed v_0. The ball lands 20 m away. Use is $\vec{a} = \langle 0,-10 \rangle$ for the acceleration due to gravity.
   a) Find the velocity vector $\vec{v}$ (Your answer can have v_0 in it)
   b) Find the position vector $\vec{r}$ (Your answer can have v_0 in it)
   c) Find the initial speed v_0



a) $\vec{v} = \langle 0, -10t \rangle + \vec{c}$

$\vec{v}(0) = \langle v_0 \cos 45°, v_0 \sin 45° \rangle$

$= \langle v_0 \frac{\sqrt{2}}{2}, v_0 \frac{\sqrt{2}}{2} \rangle$

$$\boxed{\vec{v} = \langle v_0 \sqrt{2}/2, v_0 \frac{\sqrt{2}}{2} - 10t \rangle}$$

b) $\boxed{\vec{r} = \langle v_0 \frac{\sqrt{2}}{2} t, v_0 \frac{\sqrt{2}}{2} t - 5t^2 \rangle} + \cancel{\vec{d}}$ $\vec{0}$

c) $20 = v_0 \frac{\sqrt{2}}{2} t$ $\quad v_0 \frac{\sqrt{2}}{2} t - 5t^2 = 0$

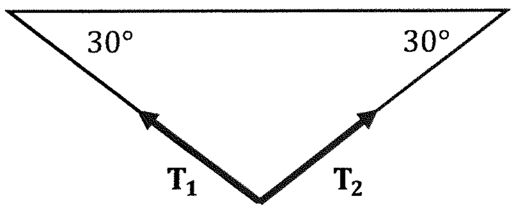
If $t=2$ $\quad 20 = v_0 \sqrt{2}$ $\quad 20 - 5t^2 = 0$

$4 = t^2$

$t_{land} = 2$

$$\boxed{v_0 = \frac{20}{\sqrt{2}}}$$

5. (15 points) A cable is suspended as shown below. It makes an angle of 30° on either side with the horizontal. If $\|\mathbf{T_1}\|=\|\mathbf{T_2}\|=12$ lbs
   a) Write tension vector $\mathbf{T_1}$ in its component form.
   b) Find the weight of the cable



a) $\vec{T_1} = \langle -12\cos 30°, 12\sin 30° \rangle$

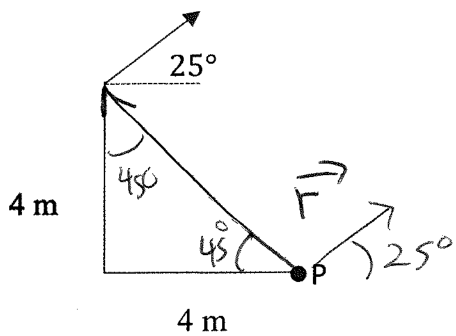
$\boxed{\vec{T_1} = \langle -12(\sqrt{3}/2), 12(1/2) \rangle}$

b) $\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$

$\vec{T_2} = \langle 12\left(\frac{\sqrt{3}}{2}\right), 12(1/2) \rangle$

$\text{weight} = 12(1/2) + 12(1/2) = \boxed{12 \text{ lb}}$

6. (12 points) Find the magnitude of the torque about P if a 24 N force is applied as shown



$$\|\vec{r}\| = \sqrt{16+16}$$

$$\theta = 110^\circ$$

$$\|\vec{\tau}\| = \|\vec{r}\|\,\|\vec{F}\|\sin\theta$$

$$\boxed{\sqrt{32}\,(24)\sin 110^\circ}$$