MA 242 Test 1 Version 1

Name: _____ (print your first and last name neatly)

1. (33 points) Use points A (1,2,4) and B (1,3,6) and vectors $\vec{b} = \hat{\mathbf{i}} + 2\hat{\mathbf{j}}$ and $\vec{c} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ to answer the following:

   a) Find the distance from point B to the xz plane

   b) Find the distance from point B to the z-axis

   c) Find two vectors parallel to $\vec{b}$, but with magnitude of 6

   d) Find the vector equation of the line segment from point A to point B

   e) Find the volume of the parallelepiped with adjacent edges $\overrightarrow{AB}$, $\vec{b}$, and $\vec{c}$

2. (15 points) Use intersecting lines L_1: x=1+t, y=3, z= 8-t and L_2: x=3s, y=1+2s, z= 7-s to answer the following:

a) Find the point of intersection of the lines L_1 and L_2

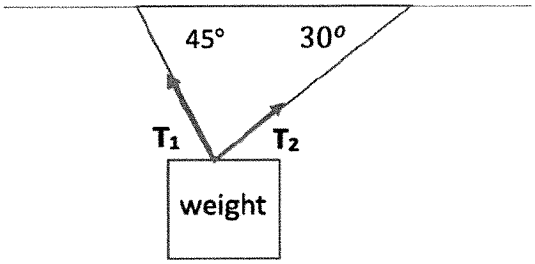
b) Find the equation of the plane containing lines L_1 and L_2

3. (18 points) A ball is thrown from the ground at an angle of elevation of 30° above the horizontal with an initial speed of 60 m/s. Use is $\vec{\mathbf{a}} = \langle 0, -10 \rangle$ for the acceleration due to gravity.
   a) Find the velocity vector $\vec{\mathbf{v}}$

   b) Find the position vector $\vec{\mathbf{r}}$

   c) Find the location of the ball at its maximum height.

4. (15 points) Use the picture to answer the following:

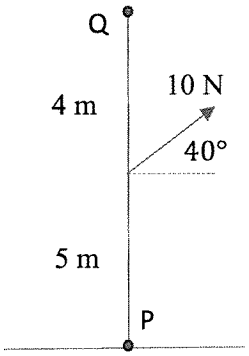


a) The magnitude of tension vector $\mathbf{T_1}$ is 30 lb. Write tension vector $\mathbf{T_1}$ in its component form

b) Find the magnitude of tension vector $\mathbf{T_2}$

c) Find the weight of the object suspended by these cables

5. (10 points) Use the image below to answer the following questions:



a) Find the magnitude of the torque about point P if a 10 N force is applied as shown.

b) Find the magnitude of the torque about point Q if a 10 N force is applied as shown

6. (9 points) Find the angle between the vectors $\mathbf{a}=\langle 4,1,1\rangle$ and $\mathbf{b}=\langle -2,1,3\rangle$

C3 T1 V1 Solutions

1. (33 points)

a) $\boxed{3}$

b) z-axis $x=0, y=0$ $\sqrt{(1-0)^2+(3-0)^2+(6-6)^2}$

$\boxed{\sqrt{10}}$

c) $\vec{b} = \langle 1,2,0 \rangle$

$\|\vec{b}\| = \sqrt{5}$

$\pm 6 \dfrac{\vec{b}}{\|\vec{b}\|} = \boxed{\pm \dfrac{6}{\sqrt{5}} \langle 1,2,0 \rangle}$

$= \pm \dfrac{\langle 6, 12, 0 \rangle}{\sqrt{5}}$

d) $\overrightarrow{AB} = \langle 1-1, 3-2, 6-4 \rangle = \langle 0,1,2 \rangle$

$\boxed{\vec{r}(t) = \langle 1,2,4 \rangle + \langle 0,1,2 \rangle t \quad 0 \le t \le 1}$

e) $\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 0 \\ -3 & 1 & 1 \end{vmatrix} = \langle 2, -(1-0), 1+6 \rangle$

$= \langle 2, -1, 7 \rangle$

$V = \langle 0,1,2 \rangle \cdot \langle 2,-1,7 \rangle = 0 - 1 + 14 = \boxed{13}$

2. (15 points)

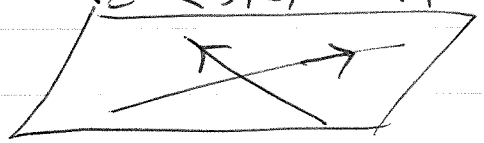
a) $1+t = 3s$

$3 = 1+2s \rightarrow s=1$ ↑ $t=2$

$8-t = 7-s$ $\qquad 6=6$ ✓

pt $\boxed{(3,3,6)}$

b) $\vec{v_2} = \langle 3,2,-1 \rangle$ $\vec{v_1} = \langle 1,0,-1 \rangle$



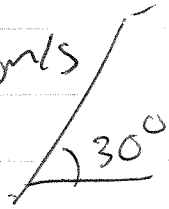
$$\vec{n} = \vec{v_1} \times \vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 3 & 2 & -1 \end{vmatrix}$$

$= \langle 0+2, -(-1+3), 2 \rangle$

$= \langle 2, -2, 2 \rangle$

$\boxed{2(x-3) - 2(y-3) + 2(z-6) = 0}$

3. (18 points)



a) $\vec{v} = \langle 0, -10t \rangle + \vec{C}$

$\vec{v}(0) = \langle 60\cos 30^\circ, 60\sin 30^\circ \rangle$

$= \langle 60\,\sqrt{3}/2, 60\,1/2 \rangle$

$= \langle 30\sqrt{3}, 30 \rangle$

$\boxed{\vec{v} = \langle 30\sqrt{3}, 30 - 10t \rangle}$

b) $\boxed{\vec{r} = \langle 30\sqrt{3}\,t, 30t - 5t^2 \rangle + \vec{A}}$

c) $30 - 10t = 0 \rightarrow t = 3$

$\boxed{\vec{r}(3) = \langle 30\sqrt{3} \cdot 3, 30(3) - 5 \cdot 3^2 \rangle}$

4. (15 points)

a) $\vec{T_1} = \langle -30\cos 45^\circ, 30\sin 45^\circ \rangle$

$= \langle -30\sqrt{2}/2, 30\sqrt{2}/2 \rangle$

$\boxed{\vec{T_1} = \langle -15\sqrt{2}, 15\sqrt{2} \rangle}$

b) $\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$

$\langle -15\sqrt{2}, 15\sqrt{2} \rangle + \langle T_2\cos 30^\circ, T_2 \sin 30^\circ \rangle$

$= \langle 0, \text{weight} \rangle$

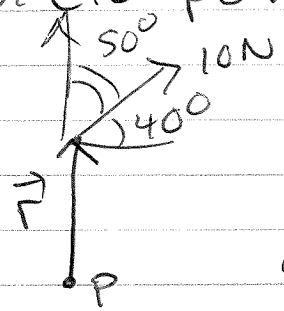
$-15\sqrt{2} + T_2 \cos 30^\circ = 0$

$-15\sqrt{2} + T_2 \sqrt{3}/2 = 0$

$$\boxed{T_2 = \frac{15\sqrt{2}}{(\sqrt{3}/2)}}$$

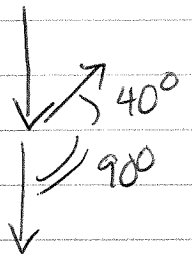
c) $\text{weight} = \boxed{15\sqrt{2} + \frac{15\sqrt{2}}{\sqrt{3}/2}\left(\frac{1}{2}\right)}$

5. (10 points)



a) $\|\vec{F}\| = 5 \cdot 10 \cdot \sin 50°$

b) $\|\vec{F}\| = 4 \cdot 10 \sin 130°$



6. (9 points)

$\vec{a} \cdot \vec{b} = \|\vec{a}\| \|\vec{b}\| \cos\theta$

$\langle 4,1,1 \rangle \cdot \langle -2,1,3 \rangle = \sqrt{18}\ \sqrt{4+1+9} \cos\theta$

$-8 + 1 + 3 = \sqrt{18}\sqrt{14} \cos\theta$

$$\theta = \cos^{-1}\left(\frac{-4}{\sqrt{18}\sqrt{14}}\right)$$