MA 242 Test 1 Version 2

Name: Solutions (print your first and last name neatly)

1. (33 points) Use points A (1,2,4) and B (1,4,5) and vectors $\vec{b} = 2\hat{\mathbf{i}} + \hat{\mathbf{j}}$ and $\vec{c} = -3\hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ to answer the following:

a) Find the distance from point B to the xz plane

$$\boxed{4}$$

b) Find the distance from point B to the z-axis

$$\sqrt{1^2+4^2} = \sqrt{17}$$

c) Find two vectors parallel to $\vec{b}$, but with magnitude of 4

$$\|\vec{b}\| = \sqrt{5} \qquad \pm \frac{4\vec{b}}{\|\vec{b}\|} = \boxed{\pm\frac{4}{\sqrt{5}}\langle 2,1,0\rangle}$$

d) Find the vector equation of the line segment from point A to point B

$$\vec{AB} = \langle 1-1, 4-2, 5-4\rangle = \langle 0,2,1\rangle$$

$$\boxed{\vec{r} = \langle 1,2,4\rangle + \langle 0,2,1\rangle t \qquad 0 \le t \le 1}$$

e) Find the volume of the parallelepiped with adjacent edges $\overrightarrow{AB}$, $\vec{b}$, and $\vec{c}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 0 \\ -3 & 1 & 1 \end{vmatrix} = \langle 1, -(2-0), 2+3\rangle$$

$$= \langle 1, -2, 5\rangle$$

$$\vec{AB} \circ \langle 1,-2,5\rangle$$

$$\langle 0,2,1\rangle \circ \langle 1,-2,5\rangle = 0 - 4 + 5 = \boxed{1}$$

2. (15 points) Use intersecting lines L_1: $x=1+t$, $y=3$, $z=8-t$ and L_2: $x=3s$, $y=1+2s$, $z=7-s$ to answer the following:

a) Find the point of intersection of the lines L_1 and L_2

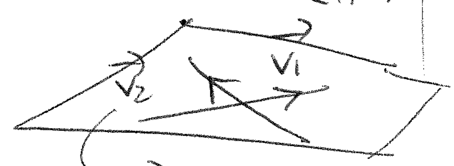
$$1+t=3s \qquad t=2$$
$$3=1+2s \longrightarrow s=1$$
$$8-t=7-s \qquad 6=6 \checkmark$$

pt!

$L_2(1)$: $\boxed{(3,3,6)}$

b) Find the equation of the plane containing lines L_1 and L_2

$\vec{v_1} = \langle 1,0,-1\rangle$



$\vec{v_2} = \langle 3,2,-1\rangle$

$$\vec{v_1}\times\vec{v_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & -1 \\ 3 & 2 & -1 \end{vmatrix}$$

$$= \langle 0+2, -(-1+3), 2-0\rangle = \langle 2,-2,2\rangle$$

$$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$$

$$\boxed{2(x-3)-2(y-3)+2(z-6)=0}$$

3. (18 points) A ball is thrown from the ground at an angle of elevation of 30° above the horizontal with an initial speed of 80 m/s. Use is $\vec{\mathbf{a}} = \langle 0,-10\rangle$ for the acceleration due to gravity.

a) Find the velocity vector $\vec{\mathbf{v}}$

$$\vec{v} = \langle 0, -10t\rangle + \vec{c}$$

$$\vec{v}(0) = \vec{c} = \langle 80\cos 30°, 80\sin 30°\rangle$$

$$= \langle 80\,\sqrt{3}/2, 80\,\tfrac{1}{2}\rangle$$

$$= \langle 40\sqrt{3}, 40\rangle$$

$$\boxed{\vec{v} = \langle 40\sqrt{3}, 40-10t\rangle}$$

b) Find the position vector $\vec{\mathbf{r}}$

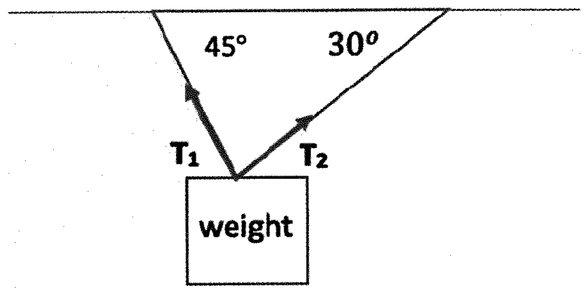
$$\boxed{\vec{r} = \langle 40\sqrt{3}\,t, 40t - 5t^2\rangle} + \cancel{\vec{d}} \; \vec{0}$$

c) Find the location of the ball at its maximum height.

$$40 - 10t = 0 \rightarrow t = 4$$

$$\boxed{\vec{r}(4) = \langle 40\sqrt{3}(4), 40(4) - 5\cdot 4^2\rangle}$$

4. (15 points) Use the picture to answer the following:



a) The magnitude of tension vector $\mathbf{T_1}$ is 20 lb. Write tension vector $\mathbf{T_1}$ in its component form

$$\vec{T_1} = \langle -20\cos 45^\circ, 20\sin 45^\circ \rangle$$
$$= \langle -20\sqrt{2}/2, 20\sqrt{2}/2 \rangle$$
$$= \langle -10\sqrt{2}, 10\sqrt{2} \rangle$$

b) Find the magnitude of tension vector $\mathbf{T_2}$

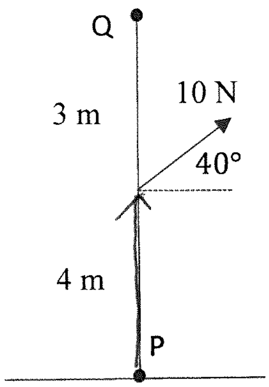
$$\vec{T_1} + \vec{T_2} = \langle 0, \text{weight} \rangle$$
$$\langle \underline{-10\sqrt{2}}, 10\sqrt{2} \rangle + \langle \underline{T_2\cos 30^\circ}, T_2 \sin 30^\circ \rangle = \langle 0, w \rangle$$
$$-10\sqrt{2} + T_2 \cos 30^\circ = 0$$
$$-10\sqrt{2} + T_2 \sqrt{3}/2 = 0 \qquad T_2 = \boxed{\frac{10\sqrt{2}}{\sqrt{3}/2}}$$

c) Find the weight of the object suspended by these cables

$$\boxed{\text{Weight} = 10\sqrt{2} + \frac{10\sqrt{2}}{\sqrt{3}/2}\,\frac{1}{2}}$$

$\sin 30^\circ$

5. (10 points) Use the image below to answer the following questions:



a) Find the magnitude of the torque about point P if a 10 N force is applied as shown.

$$\|\vec{\tau}\| = 4 \cdot 10 \sin 50^\circ$$

b) Find the magnitude of the torque about point Q if a 10 N force is applied as shown

$$\|\vec{\tau}\| = 3 \cdot 10 \sin(130^\circ)$$

6. (9 points) Find the angle between the vectors **a**=<3,1,1> and **b**=<-1,2,2>

$$\vec{a} \circ \vec{b} = \|\vec{a}\| \, \|\vec{b}\| \cos\theta$$

$$\langle 3,1,1 \rangle \circ \langle -1,2,2 \rangle = \sqrt{11} \quad \sqrt{1+4+4} \cos\theta$$

$$-3+2+2 = \sqrt{11}\,\sqrt{9} \cos\theta$$

$$\boxed{\theta = \cos^{-1}\left(\frac{1}{\sqrt{11}\,\sqrt{9}}\right)}$$