C3 T2 V1 Solutions

1. (17 points)

$f_x = -x^2 + 4y = 0$

$f_y = 4x - 4y = 0 \rightarrow x = y$

$-x^2 + 4x = 0$

$x(-x+4) = 0$

$x = 0, \; x = 4$

$(0,0)$ & $(4,4)$

$f_{xx} = -2x \quad f_{yy} = -4 \quad f_{xy} = 4$

$D = f_{xx} f_{yy} - [f_{xy}]^2$

$= 8x - 16$

$D(0,0) = -16 < 0 \rightarrow (0,0) =$ saddle pt

$D(4,4) = 32 - 16 > 0$

$f_{xx}(4,4) = -8 < 0$

$\cap$

$(4,4) =$ local max

2. (20 points)

a) $\nabla f = \langle f_x, f_y \rangle$

$= \langle 6xy\sec^2(3x^2y-6), 3x^2\sec^2(3x^2y-6) \rangle$

$\nabla f(1,2) = \langle 12\sec^2 0, 3\sec^2 0 \rangle$

$= \langle 12, 3 \rangle$

$\overrightarrow{PQ} = \langle 4-1, 6-2 \rangle = \langle 3, 4 \rangle$

$\|\overrightarrow{PQ}\| = \sqrt{9+16} = 5$ $\quad \hat{u} = \langle \frac{3}{5}, \frac{4}{5} \rangle$

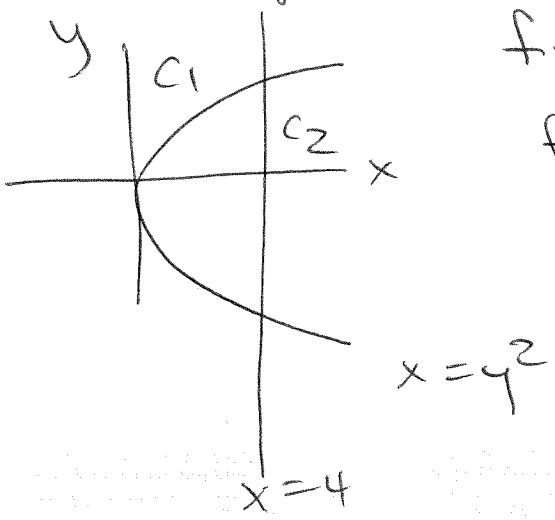
$D_{\hat{u}} f = \nabla f \cdot \hat{u}$

$= \langle 12, 3 \rangle \cdot \langle \frac{3}{5}, \frac{4}{5} \rangle$

$= \frac{36}{5} + \frac{12}{5} = \frac{48}{5}$

b) $\nabla f = \langle 12, 3 \rangle$

3. (22 pts)



$f_x = 2x = 0$
$f_y = -4y = 0$
} critical pt (0,0)

Candidates

$f(0,0) = 0$
$f(0,0) = 0$
$f(1,1) = -1$
$f(1,-1) = -1$
$f(4,0) = 16$
$f(4,2) = 16 - 8 = 8$
$f(4,-2) = 8$

C_1: $x = y^2$: $f(y^2, y) = y^4 - 2y^2$

$f'(y^2, y) = 4y^3 - 4y$

$= 4y(y^2 - 1)$

$y = 0, \quad y = \pm 1$

$(0,0), (1,1), (1,-1)$

C_2: $x = 4$ $f(4,y) = 16 - 2y^2$

$f'(4,y) = -4y = 0$

$y = 0$

$(4,0)$

endpts: $4 = y^2 \rightarrow y = \pm 2$

$(4,2)$ $(4,-2)$

Global max = 16 Global min = -1

4. (14 points)

$\nabla f = \langle 2, 6 \rangle = \lambda \nabla g = \lambda \langle 2x, 2y \rangle$

$2 = \lambda 2x \qquad 6 = \lambda 2y$

$x = 1/\lambda \qquad y = 3/\lambda \qquad \lambda \neq 0$ since $2 \neq 0 \cdot 2x = 0$

$x^2 + y^2 = 10$

$\left(\frac{1}{\lambda}\right)^2 + \left(\frac{3}{\lambda}\right)^2 = 10$

$\frac{1}{\lambda^2} + \frac{9}{\lambda^2} = 10$

$\frac{10}{\lambda^2} = 10 \qquad \lambda = \pm 1$

$\lambda = 1 \rightarrow x = 1, y = 3$

$\lambda = -1 \rightarrow x = -1, y = -3$

$f(1,3) = 2 + 18 = 20 \leftarrow \text{max}$

$f(-1,-3) = -2 - 18 = -20 \leftarrow \text{min}$

5. (13 points)

$$\vec{r}' = \langle 12, 8\tfrac{3}{2}t^{1/2}, 6t \rangle$$

$$= \langle 12, 12\sqrt{t}, 6t \rangle$$

$$L = \int_0^1 \|\vec{r}'\| \, dt$$

$$= \int_0^1 \sqrt{144 + 144t + 36t^2} \, dt$$

$$= \int_0^1 \sqrt{(12 + 6t)^2}$$

$$= \int_0^1 (12 + 6t) \, dt = 12t + 3t^2 \Big|_0^1$$

$$= \boxed{15}$$

6. (14 points)

$$\frac{dz}{dt} = \frac{dz}{dx}\frac{dx}{dt} + \frac{dz}{dy}\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{2}(2x+6y)^{-1/2} \cdot 2\,(2t) + \frac{1}{2}(2x+6y)^{-1/2} 6\,(st^{s-1})$$

$$\frac{dz}{dt} = \frac{2t}{\sqrt{2(s+t^2)+6t^s}} + \frac{3st^{s-1}}{\sqrt{2(s+t^2)+6t^s}}$$