MA 242 Test 2 Version 2

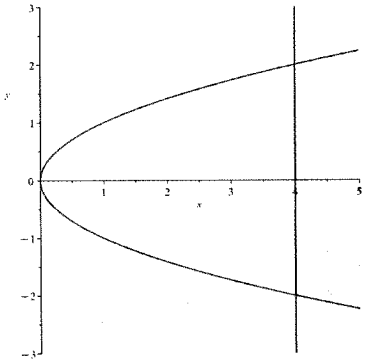
Name: _____ (print your first and last name neatly)

1. (17 points) Find and classify all critical points of f(x,y)= $-\frac{x^3}{3} + 2xy - y^2$ Justify your answers as we have done in class.

2. (20 points) Use the function f(x,y)=tan($xy^2 - 2$) to find the following:
   a) Find the directional derivative of f(x,y) at P(2,1) in the direction of the point Q(6,4)

   b) In what direction does f(x,y) increase most rapidly at P(2,1)?

3. (22 points) Find the global maximum and minimum values of f(x,y)=$x^2 - 2y^2$ on the region D bounded by x=4 and the curve $x = y^2$ shown below. Fully justify your answers as we have done in class.



4. (14 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)= $6x + 2y$ subject to the given constraint, $x^2 + y^2 = 10$

5. (13 points) Find the length of the curve given by $\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$ where $0 \leq t \leq 1$

6. (14 points) Given $z=\sqrt{4x+2y}$, where $x=s+t^3$, $y=t^s$. Use the Calculus 3 Chain Rule to first write a formula for $\frac{\partial z}{\partial t}$ and then find it.

MA 242 Test 2 Version 2

Name: Solutions (print your first and last name neatly)

1. (17 points) Find and classify all critical points of f(x,y)= $-\frac{x^3}{3} + 2xy - y^2$
Justify your answers as we have done in class.

$f_x = -x^2 + 2y$ $\qquad -x^2 + 2x = 0$

$f_y = 2x - 2y = 0 \rightarrow x = y$ $\qquad x(-x+2) = 0$

$x = 0 \quad x = 2$

critical pts: $(0,0), (2,2)$

$f_{xx} = -2x$

$f_{yy} = -2$

$f_{xy} = 2$

$D = f_{xx} f_{yy} - [f_{xy}]^2 = 4x - 4$

$D(0,0) = -4 < 0$ $\qquad (0,0) =$ saddle pt

$D(2,2) = 8 - 4 > 0$

$f_{xx}(2,2) = -4 < 0$

$(2,2) =$ local max



2. (20 points) Use the function f(x,y)=tan($xy^2 - 2$) to find the following:
   a) Find the directional derivative of f(x,y) at P(2,1) in the direction of the point Q(6,4)

$\nabla f = \langle f_x, f_y \rangle$

$= \langle y^2 \sec^2(xy^2-2), 2xy\sec^2(xy^2-2) \rangle$

$\nabla f(2,1) = \langle 1\sec^2 0, 4\sec^2 0 \rangle = \langle 1, 4 \rangle$

$\overrightarrow{PQ} = \langle 6-2, 4-1 \rangle = \langle 4, 3 \rangle$ $\quad \|\overrightarrow{PQ}\| = 5$

$\hat{u} = \dfrac{\overrightarrow{PQ}}{\|\overrightarrow{PQ}\|} = \dfrac{\langle 4,3 \rangle}{5} = \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle$

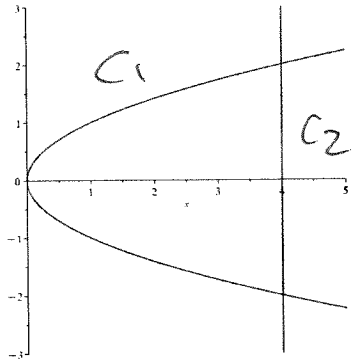
$D_{\hat{u}} f = \nabla f \cdot \hat{u} =$

$\langle 1,4 \rangle \cdot \left\langle \frac{4}{5}, \frac{3}{5} \right\rangle = \boxed{\frac{4}{5} + \frac{12}{5}} = \frac{16}{5}$

   b) In what direction does f(x,y) increase most rapidly at P(2,1)?

$\nabla f = \langle 1, 4 \rangle$

3. (22 points) Find the global maximum and minimum values of $f(x,y)=x^2-2y^2$ on the region D bounded by x=4 and the curve $x=y^2$ shown below. Fully justify your answers as we have done in class.



$f_x = 2x = 0$
$f_y = -4y = 0$ $\rightarrow (0,0)$

C_1: $f(y^2,y) = y^4 - 2y^2$

$f'(y^2,y) = 4y^3 - 4y = 0$

$4y(y^2-1) = 0$

$y = 0, y = \pm 1$

$x = y^2$: $(0,0)$, $(1,1)$, $(1,-1)$

C_2: $x = 4$: $f(4,y) = 16 - 2y^2$

$f'(4,y) = -4y = 0 \rightarrow y = 0$

$f(4,0)$

endpoints: $4 = y^2 \rightarrow y = \pm 2$

Candidates

$f(0,0) = 0$
$f(0,0) = 0$
$f(1,1) = -1$
$f(1,-1) = -1$
$f(4,0) = 16$
$f(4,2) = 8$
$f(4,-2) = 8$

Global max value = 16

Global min value = -1

4. (14 points) Use Lagrange multipliers to find the maximum and minimum values of the function f(x,y)= $6x + 2y$ subject to the given constraint, $x^2 + y^2 = 10$

$\nabla f = \langle f_x, f_y \rangle = \langle 6,2 \rangle = \lambda \nabla g = \lambda \langle 2x, 2y \rangle$

$6 = \lambda 2x$ $\quad x = 3/\lambda$ $\quad (\lambda \neq 0$ since $\lambda = 0 \rightarrow 6 = 0)$

$2 = \lambda 2y$ $\quad y = \frac{1}{\lambda}$

$x^2 + y^2 = 10$

$\frac{9}{\lambda^2} + \frac{1}{\lambda^2} = 10$

$\frac{10}{\lambda^2} = 10$ $\quad \lambda^2 = 1$ $\quad \lambda = \pm 1$

max

$\lambda = 1 \rightarrow x = 3, y = 1$ $\quad f(3,1) = 18 + 2 = 20$

$\lambda = -1 \rightarrow x = -3, y = -1$ $\quad f(-3,-1) = -18 - 2 = -20$

min

5. (13 points) Find the length of the curve given by $\vec{r}(t) = \langle 12t, 8t^{3/2}, 3t^2 \rangle$ where $0 \leq t \leq 1$

$$\vec{r}\,' = \langle 12, 8\tfrac{3}{2}t^{1/2}, 6t \rangle$$

$$= \langle 12, 12\sqrt{t}, 6t \rangle$$

$$L = \int_a^b \|\vec{r}\,'\| \, dt$$

$$= \int_0^1 \sqrt{144 + 144t + 36t^2} \, dt$$

$$= \int_0^1 \sqrt{(12+6t)^2} \, dt$$

$$= \int_0^1 12 + 6t \, dt = 12t + 3t^2 \Big|_0^1$$

$$= \boxed{15}$$

6. (14 points) Given $z=\sqrt{4x+2y}$, where $x=s+t^3$, $y=t^s$. Use the Calculus 3 Chain Rule to first write a formula for $\frac{\partial z}{\partial t}$ and then find it.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

$$\frac{dz}{dt} = \frac{1}{2}(4x+2y)^{-1/2} \cdot 4\left(3t^2\right) + \frac{1}{2}(4x+2y)^{-1/2} \cdot 2\left(st^{s-1}\right)$$

$$\frac{dz}{dt} = \frac{6t^2}{\sqrt{4(s+t^3)+2t^s}} + \frac{st^{s-1}}{\sqrt{4(s+t^3)+2t^s}}$$