MA 242 Test 3 Version 1

Name: _____ (Please print neatly)

1. (15 points) Evaluate the iterated integral by switching the order of integration $\int_0^4 \int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dy\, dx$

2. (22 points) F is the solid bounded by the plane z=32 and the paraboloid $z = 2x^2 + 2y^2$ with density $\sigma(x, y, z) = x^4 z$

   a) Set up an iterated triple integral as an x-simple region to find the mass of F. **Do not evaluate!**

   b) Set up an iterated integral using cylindrical coordinates to find the mass of F. **Do not evaluate!**

3. (15 points) Find the average value of f(x,y)=$\frac{2y}{6-3x}$ over the triangular lamina with vertices (0,0), (0,6), and (2,0)

4. (20 points) Use D=$\{(x,y)|\ x^2+y^2 \leq 4,\ 0 \leq y \leq x\}$ to answer the following:

a) Find the mass of the lamina occupying the region D with density $\sigma(x,y)$=arctan$\left(\frac{y}{x}\right)$

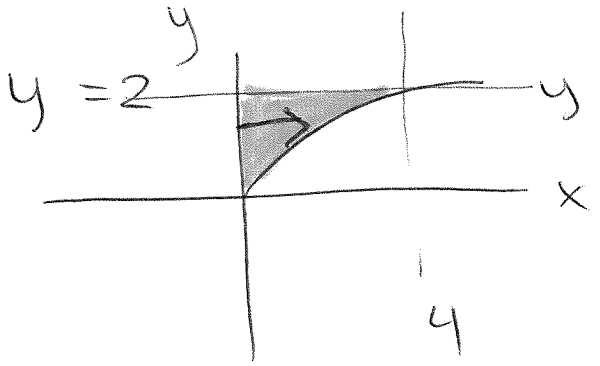
b) Set up the iterated integrals in polar coordinates needed to find the center of mass of D. **Do not evaluate!**

5. (16 points) **Set up** the double integral needed to find the volume of solid F that lies inside the ellipsoid $4x^2 + 4y^2 + z^2 = 36$ and outside the cylinder $x^2 + y^2 = 1$ **Do not evaluate.**

6. (12 points) Evaluate $\iiint_F \frac{e^{2x}}{x\sqrt{x^2+y^2}}\,dV$ where F is the solid bounded above by the cone $z=\sqrt{x^2+y^2}$ and below by the region in the xy-plane bounded by y=x, y=3x, and x=4.
Hint: If the integration isn't relatively easy, you are doing something wrong.

C3 T3 V1 Solutions

1. (15 points)



$$\int_0^2 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy$$

$$\int_0^2 \frac{x}{y^3+1}\Big|_0^{y^2} dy$$

$$\int_0^2 \frac{y^2}{y^3+1}\, dy \qquad u = y^3+1$$

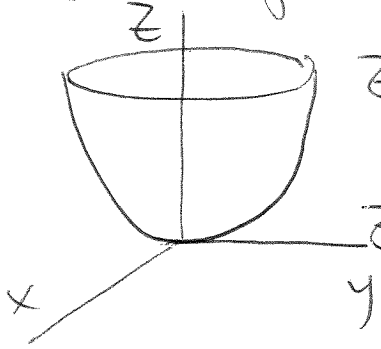
$$du = 3y^2\, dy$$

$$\frac{1}{3}du = y^2\, dy$$

$$\int_1^9 \frac{1}{3}\cdot\frac{1}{u}\, du = \frac{1}{3}\ln u\Big|_1^9$$

$$\frac{1}{3}(\ln 9 - \cancel{\ln 1}^{0})$$

2. (22 points)

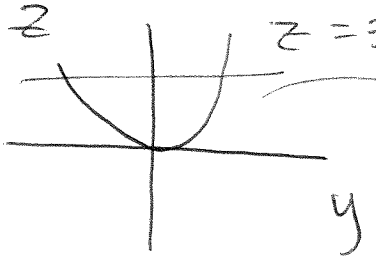


$z = 2x^2 + 2y^2 \rightarrow z - 2y^2 = 2x^2$

$x^2 = \frac{1}{2}z - y^2$

$x = \pm\sqrt{\frac{1}{2}z - y^2}$

a) $\int_{-4}^{4} \int_{2y^2}^{32} \int_{-\sqrt{\frac{1}{2}z-y^2}}^{\sqrt{\frac{1}{2}z-y^2}} x^4 z \, dx \, dz \, dy$

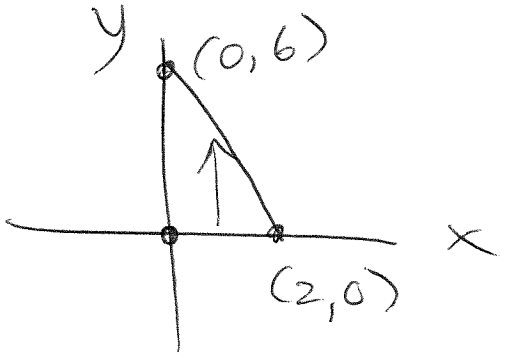


$32 = 2y^2 \quad 16 = y^2$

b) $\int_0^{2\pi} \int_0^4 \int_{2r^2}^{32} (r\cos\theta)^4 z \, r \, dz \, dr \, d\theta$

$32 = 2r^2 \quad 16 = r^2$

3. (15 points)



$$m = \frac{\Delta y}{\Delta x} = \frac{0-6}{2-0} = -3$$

$$y = 6 - 3x$$

$$\int_0^2 \int_0^{6-3x} \frac{2y}{6-3x}\, dy\, dx$$

$$\int_0^2 \frac{y^2}{6-3x}\Big|_0^{6-3x} dx$$

$$\int_0^2 \frac{(6-3x)^2}{6-3x}\, dx$$

$$\int_0^2 6-3x\, dx = 6x - \frac{3}{2}x^2\Big|_0^2$$

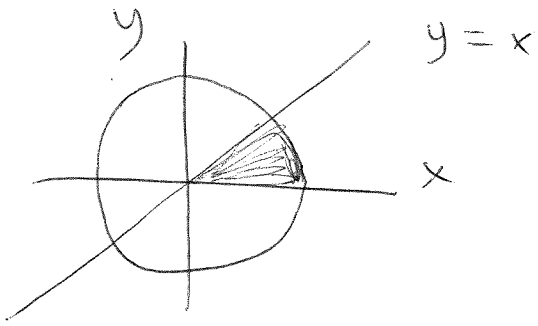
$$12 - 6 = \boxed{6}$$

$$\frac{\int_0^2 \int_0^{6-3x} \frac{2y}{6-3x}\, dy\, dx}{\frac{1}{2}(6)(2)} = \frac{6}{6} = \boxed{1}$$

↑ average value

or $\int_0^2 \int_0^{6-3x} 1\, dy\, dx = \ldots$

4.



a)

$$m = \int_0^{\pi/4}\int_0^2 \arctan\left(\frac{\sin\theta}{\cos\theta}\right) r\,dr\,d\theta \qquad \tan\theta$$

$$\int_0^{\pi/4}\int_0^2 \theta\, r\,dr\,d\theta$$

$$\int_0^{\pi/4} \theta\,d\theta \int_0^2 r\,dr$$

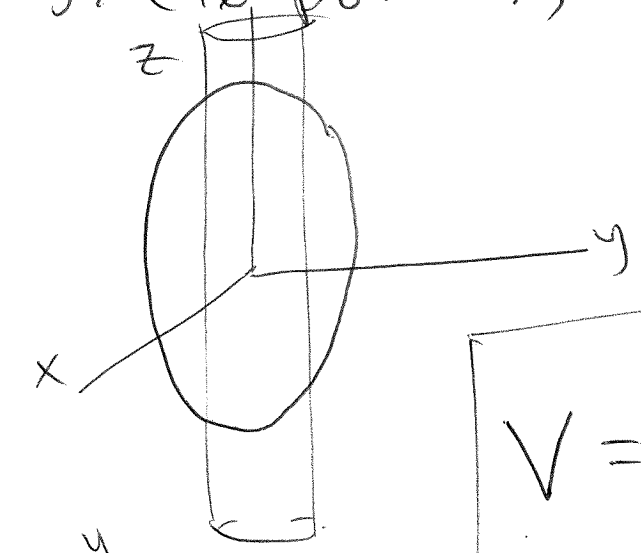
$$\tfrac{1}{2}\theta^2\Big|_0^{\pi/4} \quad \tfrac{1}{2}r^2\Big|_0^2$$

$$\boxed{\tfrac{1}{2}\left(\tfrac{\pi}{4}\right)^2\left(\tfrac{1}{2}\cdot 4\right)} = \left(\frac{\pi}{4}\right)^2$$

b)

$$\left(\frac{\int_0^{\pi/4}\int_0^2 r\cos\theta\;\theta\, r\,dr\,d\theta}{(\pi/4)^2},\ \frac{\int_0^4\int_0^2 r\sin\theta\;\theta\, r\,dr\,d\theta}{(\pi/4)^2}\right)$$

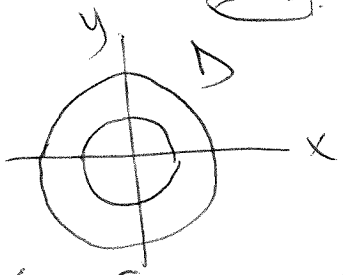
5. (16 points)



$z^2 = 36 - 4x^2 - 4y^2$

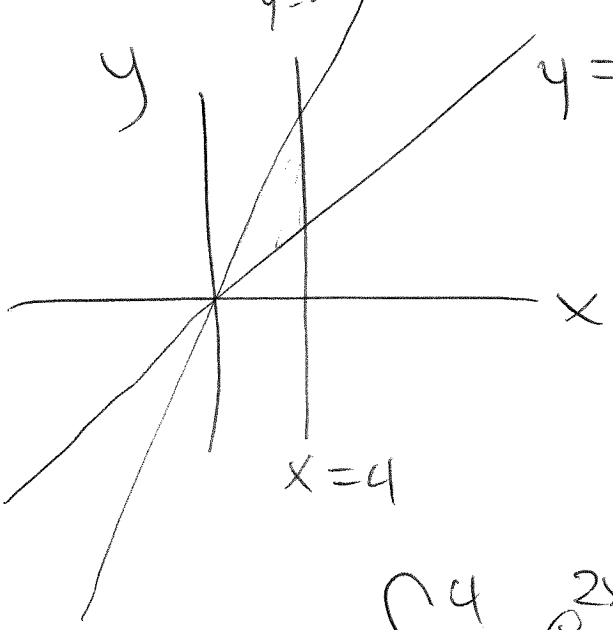
$0 = 36 - 4r^2 \quad r^2 = 9$

$$V = 2\int_0^{2\pi}\int_1^3 \sqrt{36-4r^2}\, r\, dr\, d\theta$$



6. (12 points)

$$\int_0^4 \int_x^{3x} \int_0^{\sqrt{x^2+y^2}} \frac{e^{2x}}{x\sqrt{x^2+y^2}}\, dz\, dy\, dx$$



$$\int_0^4 \int_x^{3x} \frac{e^{2x}}{x\sqrt{x^2+y^2}} z \Big|_0^{\sqrt{x^2+y^2}} dy\, dx$$

$$\int_0^4 \int_x^{3x} \frac{e^{2x}}{x}\, dy\, dx$$

$$\int_0^4 \frac{e^{2x}}{x} y \Big|_x^{3x} dx$$

$$\int_0^4 \frac{2e^{2x}}{x}(3x - x)\, dx = e^{2x}\Big|_0^4 = \boxed{e^8 - 1}$$