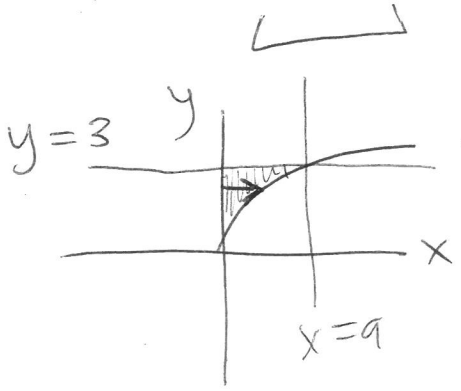
MA 242 Test 3 Version 2

Name: Solutions (Please print neatly)

1. (15 points) Evaluate the iterated integral by switching the order of integration $\int_0^9 \int_{\sqrt{x}}^3 \frac{1}{y^3+1}\, dy\, dx$



$y = \sqrt{x} \rightarrow x = y^2$

$$= \int_0^3 \int_0^{y^2} \frac{1}{y^3+1}\, dx\, dy$$

$$\int_0^3 \frac{x}{y^3+1} \Big|_0^{y^2} dy$$

$$\int_0^3 \frac{y^2}{y^3+1}\, dy$$

$u = y^3 + 1$

$du = 3y^2\, dy$

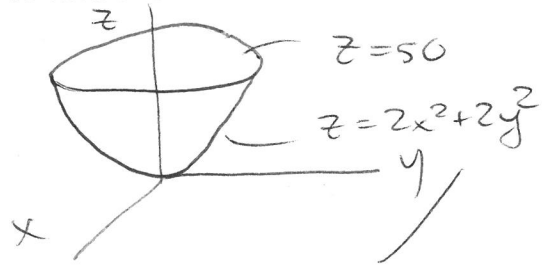
$\frac{1}{3} du = y^2\, dy$

$$\frac{1}{3} \int_1^{28} \frac{1}{u}\, du$$

$$\frac{1}{3} \ln u \Big|_1^{28} = \frac{1}{3}(\ln 28 - \cancel{\ln 1}^{0})$$

2. (22 points) F is the solid bounded by the plane z=50 and the paraboloid $z = 2x^2 + 2y^2$ with density $\sigma(x, y, z) = y^4 z$

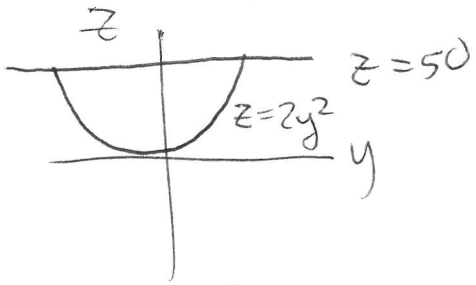
a) Set up an iterated triple integral as an x-simple region to find the mass of F. **Do not evaluate!**



$X^2 = \frac{z}{2} - y^2$

$m = \iiint_F y^4 z \, dV$

$$\int_{-5}^{5} \int_{2y^2}^{50} \int_{2y^2 - \sqrt{\frac{z}{2} - y^2}}^{\sqrt{\frac{z}{2} - y^2}} y^4 z \, dx \, dz \, dy$$



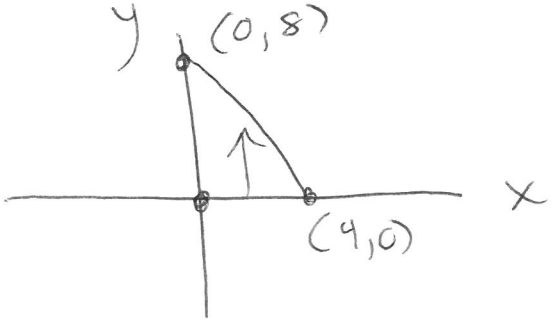
$50 = 2y^2$

$25 = y^2$

b) Set up an iterated integral using cylindrical coordinates to find the mass of F. **Do not evaluate!**

$$\int_0^{2\pi} \int_0^5 \int_{2r^2}^{50} (r\sin\theta)^4 z \, r \, dz \, dr \, d\theta$$

3. (15 points) Find the average value of $f(x,y)=\frac{2y}{8-2x}$ over the triangular lamina with vertices (0,0), (0,8), and (4,0)



$$m=\frac{\Delta y}{\Delta x}=\frac{0-8}{4-0}=-2$$

$$y=8-2x$$

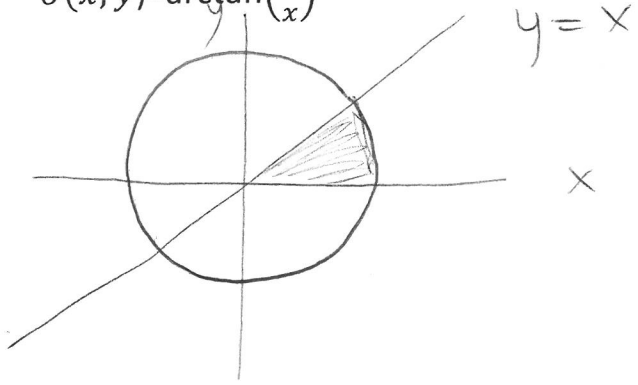
$$\frac{\iint f\,dA}{\iint 1\,dA}=\frac{\int_0^4\int_0^{8-2x}\frac{2y}{8-2x}\,dy\,dx}{\frac{1}{2}\cdot 4\cdot 8}$$

$$=\frac{\int_0^4 \frac{y^2}{8-2x}\Big|_0^{8-2x}\,dx}{16}=\frac{\int_0^4 \frac{(8-2x)^2}{8-2x}\,dx}{16}$$

$$\frac{\int_0^4 8-2x\,dx}{16}=\frac{8x-x^2\Big|_0^4}{16}=\frac{32-16}{16}=1$$

4. (20 points) Use D=$\{(x,y)|\ x^2+y^2 \le 9,\ 0 \le y \le x\}$ to answer the following:

a) Find the mass of the lamina occupying the region D with density $\sigma(x,y)=\arctan\left(\frac{y}{x}\right)$



$$m = \iint \arctan\left(\frac{y}{x}\right) dA$$

$$= \int_0^{\pi/4}\int_0^3 \arctan\left(\frac{r\sin\theta}{r\cos\theta}\right) r\,dr\,d\theta \qquad \tan\theta$$

$$\int_0^{\pi/4} \theta\,d\theta \int_0^3 r\,dr$$

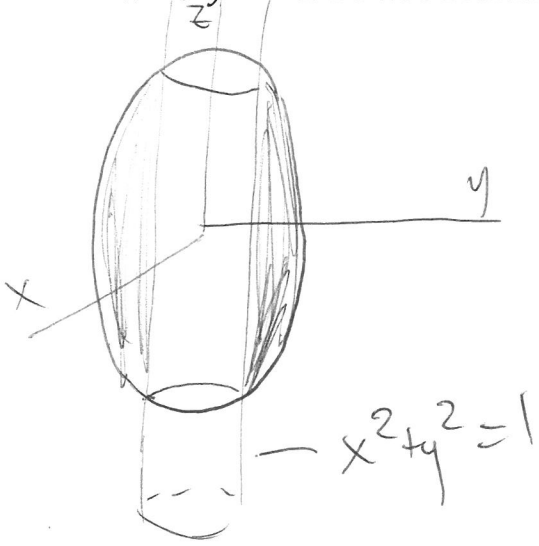
$$\frac{1}{2}\theta^2\Big|_0^{\pi/4}\ \frac{1}{2}r^2\Big|_0^3 = \boxed{\frac{1}{4}\left(\frac{\pi}{4}\right)^2 \cdot 9}$$

b) Set up the iterated integrals in polar coordinates needed to find the center of mass of D. **Do not evaluate!**

$$(\bar{x},\bar{y}) = \left(\frac{\int_0^{\pi/4}\int_0^3 (r\cos\theta)\,\theta\, r\,dr\,d\theta}{\frac{1}{4}\left(\frac{\pi}{4}\right)^2 \cdot 9}\right.$$

$$\left.\frac{\int_0^{\pi/4}\int_0^3 (r\sin\theta)\,\theta\, r\,dr\,d\theta}{\frac{1}{4}\left(\frac{\pi}{4}\right)^2 \cdot 9}\right)$$

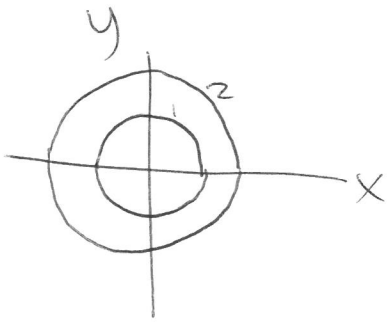
5. (16 points) **Set up** the double integral needed to find the volume of solid F that lies inside the ellipsoid $4x^2 + 4y^2 + z^2 = 16$ and outside the cylinder $x^2 + y^2 = 1$ **Do not evaluate.**



$$V = 2\int_0^{2\pi}\int_1^2 \sqrt{16-4r^2}\, r\,dr\,d\theta$$

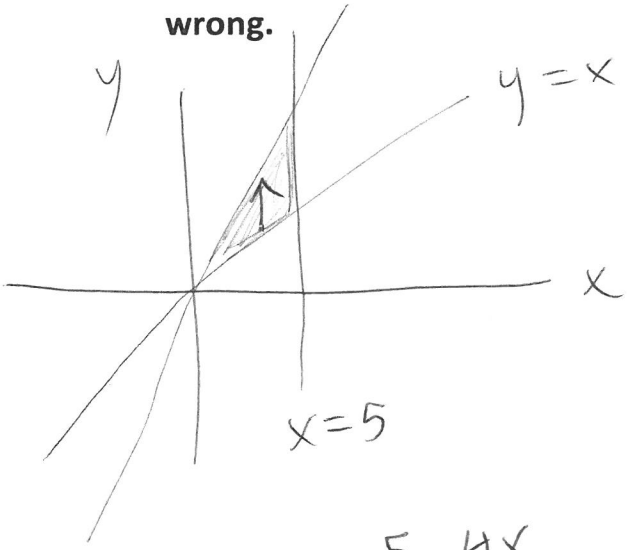
$16-4r^2=0$

$4=r^2$



6. (12 points) Evaluate $\iiint_F \frac{e^{3x}}{x\sqrt{x^2+y^2}}\,dV$ where F is the solid bounded above by the cone $z=\sqrt{x^2+y^2}$ and below by the region in the xy-plane bounded by y=x, y=4x, and x=5.

Hint: If the integration isn't relatively easy, you are doing something wrong.



$$\int_0^5 \int_x^{4x} \int_0^{\sqrt{x^2+y^2}} \frac{e^{3x}}{x\sqrt{x^2+y^2}}\,dz\,dy\,dx$$

$$= \int_0^5 \int_x^{4x} \frac{e^{3x}}{x\sqrt{x^2+y^2}}\, z \Big|_0^{\sqrt{x^2+y^2}}\,dy\,dx$$

$$\int_0^5 \int_x^{4x} \frac{e^{3x}\sqrt{x^2+y^2}}{x\sqrt{x^2+y^2}}\,dy\,dx$$

$$\int_0^5 \frac{e^{3x}}{x}\, y \Big|_x^{4x}\,dx = \int_0^5 \frac{e^{3x}(4x-x)}{x}\,dx$$

$$\int_0^5 3e^{3x}\,dx = e^{3x}\Big|_0^5 = \boxed{e^{15}-1}$$